THE OUTPUT POWER SPECTRUM PRODUCED BY CORRELATED DOUBLE SAMPLING

J. M. Pimbley † and G. J. Michon ‡

† Department of Mathematical Sciences

Rensselaer Polytechnic Institute

Troy, New York 12180-3590

‡ General Electric Corporate Research & Development Center

Schenectady, New York 12345

Abstract

Correlated double sampling provides an excellent method for the reduction of kTC noise in analog signal processing and has therefore found widespread use. One often hears the additional claim that correlated double sampling depresses low frequency noise. Assessment of this claim requires theoretical derivation of the output power of the sampling circuit in terms of the input power. We provide this derivation for two different realizations of correlated double sampling which yield precisely equivalent power spectra. We find that noise suppression at low frequency depends to a great extent on system bandwidth since the input power spectrum *aliases* to frequencies separated by multiples of the clock frequency in the output spectrum.

I. Introduction

Operation of charge-coupled devices (CCD's) and charge-injection devices (CID's) typically requires the precise measurement of a quantity of mobile charge transferred onto a capacitive node. One continually *resets* this node to a reference potential in preparation for evaluation of the next charge packet. A metal-oxide-semiconductor field-effect transistor (MOSFET) in which the gate is connected to a clock pulse serves as the switch to reset the reference node.

Unfortunately, such a reset operation is prone to *kTC noise*. The instantaneous potential difference (or, equivalently, charge) across a capacitor will fluctuate with a mean square value of kT/C (or kTC for charge) as long as there exists a source of charge such as a resistor or inverted MOSFET channel to ground. When the external source of charge is severed, as when the MOSFET is switched off, the node remains at a constant potential equal to the reference level plus the kTC noise imparted by the switch. Clearly, the existence of such noise limits the dynamic range of the analog signal processing system since the noise appears in the signal measurement.

White et. al. promulgated a circuit design solution to kTC noise fifteen years ago [1]. Their *correlated double sampling* technique essentially measures the reset level of the reference node prior to the arrival of the charge packet and subtracts this pre-charge level from the post-charge level. This method cancels the kTC noise to high precision and hence is quite useful. Upon cursory examination, it also appears that the transfer function of the correlated double sampling (CDS) circuit will suppress low frequency noise. This tentative observation is of great importance since the infamous $1/f$ noise dominates the total noise in many systems.

It appears, then, that a complete understanding of the output power spectrum of the CDS circuit for a given input power spectrum (including $1/f$ noise) would be valuable in system noise analysis and design. An additional consideration amplifies this assertion. In imaging systems, the character (i.e. spectrum) of the noise is significant beyond the simple total noise given by the integral of the noise power spectrum. Low frequency noise is more objectionable to the human eye than is high frequency noise. With little explanation, reference [1] provides a transfer function for the CDS circuit which we consider unacceptable. This inaccuracy does not detract at all from the important discovery of correlated double sampling. Nearly all subsequent attempts to elucidate the CDS output power spectrum are either incorrect or incomplete. As we shall discuss, we dispute the methods and/or results of Kansy [2] and Hopkinson and Lumb [3]. Brodersen and Emmons [4] determine the integral of the output spectrum in the special case of first-order low-pass filtered white noise at the input. Wey and Guggenbühl [5], on the other hand, provide a strong analysis. The value of our work beyond that of Wey and Guggenbühl lies mainly in the explicit demonstration that two circuit realizations of CDS give equivalent results.

In the next section we derive the CDS output power for two circuit realizations of the double sampling procedure. One realization employs two MOSFET switches while the other replaces one switch with a delay element. Though not identical in function, we find both methods produce equivalent output power spectra. For arbitrary input power, the output exhibits aliasing in multiples of the frequency with which the switch (switches) of the CDS circuit closes (close). White noise on the input, therefore, may lead to large low frequency output power spectrum values if the system bandwidth is appreciable. Section II finishes with the application of our general results to derivation of both the power spectrum and its integral for the special case of an input signal of first-order low-pass filtered white noise.

The last section discusses the results of the derivations of section II. In particular, we plot the output power spectrum for two typical cases in which the input may be described as the combination of a (white) Johnson noise and $1/f$ noise. Comparison of this effort with previously reported research of similar intent resides in section III as well. We close with a brief summary.

II. Power Spectrum Derivation

There exist two equivalent paths for the derivation of the ouput power spectrum. First, one may compute the time-dependent signal $v(t)$, determine its Fourier transform $V(f)$, which we will henceforth label the Fourier amplitude, and finally write the power spectrum $S(f)$ as the complex square of this Fourier amplitude. Second, when possible, one constructs the autocorrelation function $R(\tau) = < v^*(t)v(t+\tau) >$ and then finds the power spectrum $S(f)$ as the Fourier transform of $R(\tau)$. In addition to the choice of power spectrum calculation, one may also select one of two circuit realizations of the correlated double sampling function (see figures 1a [1,2] and 1b [2]).

Consider first the simplified circuit of figure 1a [1,2]. The input feeds through capacitor C_1 to a node isolated by switches S_1 and S_2. Switch S_1, when closed, shorts this node to ground (though it is trivial to replace ground with a non-zero bias as did White et. al. [1]). Switch S_2 leads to the output which, in turn, feeds through capacitor C_2 to ground. Omitted from this picture are amplifying stages so that we need not worry about limited charge on each node. That is, there is no capacitive division of charge so that the values of C_1 and C_2 are not important. The switches close with period T. In our formulation, each switch remains closed for a neglible fraction of the clock period. Closure of S_1 pins the intermediate node and hence essentially measures the initial kTC offset. Closure of S_2 results in signal charge measurement. S_2 closes a time λ following S_1. Of course, λ is less than T.

To derive the Fourier amplitude of the output, we write the time-varying output bias as a function of the input as

$$v_{out}(t) = \sum_{n=-\infty}^{+\infty} W(t - nT - \lambda)\, v_{mid}(nT + \lambda) \qquad (1a)$$

$$v_{mid}(t) = \sum_{m=-\infty}^{+\infty} W(t - mT)\, [v_{in}(t) - v_{in}(mT)] \qquad (1b)$$

$$v_{out}(t) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} W(nT + \lambda - mT)\, W(t - nT - \lambda)\, [v_{in}(nT + \lambda) - v_{in}(mT)] \qquad (1c)$$

where the *window function* $W(t)$ is unity for $0 < t < T$ and is zero for all other t. Equation (1a) shows that switch S_2 forces $v_{out}(t)$ to sample-and-hold $v_{mid}(t)$. Equation (1b) expresses the action of periodic resetting of $v_{mid}(t)$ to ground followed by an interval of capacitive coupling to $v_{in}(t)$. With the window function definition, we see that $W(nT + \lambda - mT)$ is just the Kronecker delta function $\delta_{n,m}$. Thus,

$$v_{out}(t) = \sum_{n=-\infty}^{+\infty} W(t - nT - \lambda)[v_{in}(nT + \lambda) - v_{in}(nT)] \quad . \qquad (2)$$

Fourier transformation for the output amplitude $V_{out}(f)$ gives

$$V_{out}(f) = \exp[-i\pi f(T + 2\lambda)]\,\frac{\sin(\pi fT)}{\pi f} \sum_{n=-\infty}^{+\infty} [v_{in}(nT + \lambda) - v_{in}(nT)]\,\exp(-i2\pi fTn) \quad . \qquad (3)$$

The summation in equation (3) appears as the difference of two discrete Fourier transforms. With the aid of the Poisson sum rule, we may express the amplitude $V_{out}(f)$ more elegantly as

$$V_{out}(f) = \exp[-i\pi f(T + 2\lambda)]\,\frac{\sin(\pi fT)}{\pi fT} \sum_{n=-\infty}^{+\infty} V_{in}(f - n/T)\{-1 + \exp[i2\pi\lambda(f - n/T)]\} \quad . \qquad (4)$$

Taking the complex square of $V_{out}(f)$, we find

$$S_{out}(f) = 4\left[\frac{\sin(\pi fT)}{\pi fT}\right]^2 \sum_{n=-\infty}^{+\infty} S_{in}(f - n/T)\,\sin^2[\pi\lambda(f - n/T)] \quad . \qquad (5)$$

Switched CDS reset

Equation (5) thus gives the output power spectrum of the correlated double sampling circuit of figure 1a. The CDS implementation of figure 1b [2] appears to be much different. The linear delay element replaces a non-linear MOSFET switch. The time-dependent signal at the middle node differs in the two CDS realizations. In the figure 1b circuit, this middle signal is $v_{mid}(t) = v_{in}(t) - v_{in}(t - \lambda)$. This linear expression contrasts markedly with that of equation (1b). Despite this inequivalence between the time-dependent signals on the middle node, the output signal $v_{out}(t)$

is identical in the figure 1a and 1b CDS representations. Hence, the output power spectra are identical.

An interesting aspect of this CDS circuit duality is that one may analyze the figure 1b implementation far more readily in terms of the autocorrelation function analysis. In fact, Kansy [2], Brodersen and Emmons [4] and Wey and Guggenbühl [5] analyze CDS in terms of the (analytically) simpler implementation of figure 1b. We find

$$R_{out}(\tau) = \sum_{n=-\infty}^{+\infty} \Delta(t - nT)\, R_{mid}(nT) \qquad (6a)$$

$$R_{mid}(\tau) = 2\, R_{in}(\tau) - R_{in}(\tau - \lambda) - R_{in}(\tau + \lambda) \qquad (6b)$$

We define the function $\Delta(t)$ so that $\Delta(0) = 1$ and Δ decays linearly to zero at $\pm T$ and is uniformly zero outside the interval $(-T, +T)$. In terms of the *mid-point* power spectrum $S_{mid}(f)$, the output spectrum is

$$S_{out}(f) = \int_{-\infty}^{+\infty} R_{out}(\tau)\, e^{-i2\pi f\tau}\, d\tau \qquad (7a)$$

$$= \sum_{n=-\infty}^{+\infty} R_{mid}(nT) \int_{-\infty}^{+\infty} \Delta(\tau - nT)\, e^{-i2\pi f\tau}\, d\tau \qquad (7b)$$

$$= T\, [\frac{\sin(\pi fT)}{\pi fT}]^2 \sum_{n=-\infty}^{+\infty} R_{mid}(nT)\, e^{-i2\pi fnT} \qquad (7c)$$

$$= [\frac{\sin(\pi fT)}{\pi fT}]^2 \sum_{n=-\infty}^{+\infty} S_{mid}(f - n/T) \qquad (7d)$$

Stepping from equation (7a) to (7d) required recognition of the (7c) infinite sum as a discrete Fourier transform resulting from sampling $R_{mid}(\tau)$. Equation (7d) gives $S_{out}(f)$ in terms of $S_{mid}(f)$ and hence isolates the effect of the *sample-and-hold* switch S_2 on the overall power spectrum. From equation (6b),

$$S_{mid}(f) = 2[1 - \cos(2\pi f\lambda)]\,S_{in}(f) \tag{8}$$

which immediately gives

$$S_{out}(f) = 2\left[\frac{\sin(\pi fT)}{\pi fT}\right]^2 \sum_{n=-\infty}^{+\infty} S_{in}(f - n/T)\,\{1 - \cos[(2\pi\lambda(f - n/T)]\} \tag{9}$$

Dehes Like Result

With a simple trigonometric identity, one finds that equations (5) and (9) are equivalent.

For further illustration of equations (5) and (9), we make a specific choice for the input power as first-order low-pass filtered white noise:

$f_c = \frac{1}{2\pi\tau_c}$; $\omega_c = \frac{1}{\tau_c}$

$2\pi\tau_c = \frac{1}{f_c}$

$$S_{in}(f) = \frac{2\tau\, S_w}{1 + (2\pi f\tau)^2} = \frac{1/\pi f_c}{1 + (f/f_c)^2} \tag{10}$$

?

in which the -3 dB frequency is $(2\pi\tau)^{-1}$. The analytical form of this input is highly amenable to contour integration techniques so that one may derive closed-form expressions. This behavior is not shared by the $1/f$ power spectrum. Evaluation of equations (5) or (9) with the input spectrum of equation (10) gives

$$S_{out}(f) = \frac{2T\,S_w}{\sinh^2(T/2\tau) + \sin^2(\pi fT)}\left[\frac{\sin(\pi fT)}{\pi fT}\right]^2$$

$$\cdot\left\{\sinh(T/2\tau)\left[\cosh(T/2\tau) - \cosh\left(\frac{2\lambda - T}{2\tau}\right)\right] + \sin^2(\pi fT)\sinh(\lambda/\tau)\right\} \tag{11}$$

Derivation of equation (11) required the identity

$$\sum_{n=-\infty}^{\infty} \frac{e^{i2\pi\lambda(x-n)/T}}{\alpha^2+(x-n)^2} = \frac{\pi}{\alpha}\left\{ e^{-2\pi\alpha\lambda/T} + \frac{\cosh(2\pi\alpha\lambda/T)[e^{-\pi\alpha}\sinh(\pi\alpha) - \sin^2(\pi x)]}{\sinh(\pi\alpha)+\sin^2(\pi x)} \right.$$

$$\left. + i\,\frac{\sinh(2\pi\alpha\lambda/T)[e^{-\pi\alpha}\sinh(\pi\alpha) - \sin^2(\pi x)]}{\sinh(\pi\alpha)+\sin^2(\pi x)} \right\} \quad . \tag{12}$$

Integration of the equation (11) result over all frequency provides a useful check on this expression for the power spectrum. With the aid of the identity

$$\int_{-\infty}^{\infty} dx \left[\frac{\sin(\pi x)}{\pi x}\right]^2 [\sinh^2(\pi\alpha) + \sin^2(\pi x)]^{-1} = [\sinh(\pi\alpha)\cosh(\pi\alpha)]^{-1} \quad , \tag{13}$$

the total power S_{tot} emerges as

$$S_{tot} = \int_{-\infty}^{\infty} dx\, S_{out}(f) = 2\,S_w\,[1 - \exp(-\lambda/\tau)] \quad . \tag{14}$$

III. Discussion

Equation (5), or equivalently equation (9), expresses the central result of this manuscript. The output power at a frequency f is a linear combination of the input power at all frequencies ν where $\nu - f$ is some integer multiple of the clock frequency T^{-1}. This *aliasing* behavior precludes discussion of a simple transfer function. Figures 2a and 2b plot both the input and output power spectra when we model the input power as a sum of *white* and $1/f$ components with two values of the system bandwidth. Our goal with this model is to investigate the impact of correlated double sampling on typical circuit noise. We observe that, due to aliasing of the *white* spectrum from higher frequencies, the output power does not go to zero at low frequency. The power at zero frequency is strongly dependent on the circuit bandwidth with larger bandwidth implying larger power. The low frequency spectrum is flat precisely because it is generated by aliasing from many other, higher frequency, components of the input power. One also finds nulls in the ouput power spectrum at non-zero multiples of the clock frequency. We have verified these characteristics experimentally. Ignoring the oscillations, the power decays at high frequency as f^{-2} multiplied by a summation of input power spectrum values. When white noise dominates the total noise power, this summation of input values approaches a constant value in the high frequency limit. (Of course, this *high frequency limit* is still contained within the system bandwidth.)

The prevalence and importance of correlated double sampling in analog signal processing lends significance to our results. Since fifteen years have elapsed since the discovery of CDS, it is surprising that the equation (5) power spectrum has never before been published for the CDS circuit implementation of figure 1a. White et. al. [1] gave a transfer function for CDS which omits the $[(\sin x)/x]^2$ term as well as the aliasing aspect (i.e. infinite summation). In the White expression, the output power vanishes at zero frequency so that it is claimed that the correlated double sampling procedure suppresses $1/f$ noise. While we agree that CDS prevents divergence of the output power at zero frequency, we claim, as depicted in figure 2, that the power is decidedly non-zero at low frequency for a broadband source.

Kansy [2] successfully derived an upper bound for the total CDS output power (integral of the power spectrum) in the special case where the input power is purely $1/f$. He did not compute the spectrum $S_{out}(f)$. In fact, we note that Kansy's equation (3) is in error and is properly given by equation (6b) in this communication. Fortunately, this error *cancels completely* when only the integrated spectrum is sought due to the Hermitian character of the autocorrelation function. Kansy's upper bound arises from an approximation of the effect of the sample-and-hold portion of the CDS circuit on the ouput power. Apparently, he did not realize, as did Brodersen and Emmons [4] and Wey and Guggenbühl [5], that the sample-and-hold has no effect on the output power. The analytical aspect of the research of Hopkinson and Lumb [6] is seriously flawed since, again, there is no mention of aliasing of the input power.

Wey and Guggenbühl [5] provide a coherent analysis of correlated double sampling. At first it appears that this study is restricted to the case in which the input power consists of first-order low-pass filtered white noise. But this is not the case. Combination of equations (5), (6), (8) and (11) of Wey and Guggenbühl will yield our equations (5) and (9). Our work differs primarily from that of Wey and Guggenbühl in the explicit inclusion of the figure 1a realization of CDS as well as figure 1b. Wey and Guggenbühl fully understood the aliasing aspect of the CDS output power as did Kansy [2].

The ouput power spectrum with first-order low-pass filtered white noise at the input of equation (11) is equivalent to that derived by Wey and Guggenbühl [5] with the exception that the latter expression is missing a factor of $[\sin(\pi f T)/\pi f]^2$. This is most likely a typographical error. Our equation (14) shows explicitly the integral of the power spectrum at the ouput of the sample-and-hold. Brodersen and Emmons [2] and Wey and Guggenbühl [5] found the same result by integrating the sample-and-hold input spectrum (of the figure 2b CDS realization) and arguing that the total power is unchanged by the sample-and-hold. As Brodersen and Emmons noted, this total output power is double the input power in the limit of infinite -3 dB frequency for the input power. Thus, the white noise power input is doubled since correlated double sampling essentially makes two measurements of the uncorrelated (white) signal.

In summary, we have derived the output power spectrum of two implementations of the correlated double sampling circuit in terms of an arbitrary input power spectrum. We plotted this output power for an input spectrum typical of circuit noise and contrasted our results with previous efforts in this field.

ACKNOWLEDGEMENT

We are pleased to acknowledge that incisive and constuctive comments of Michael P. Weir and Dr. Jerome J. Tiemann have strengthened this manuscript.

REFERENCES

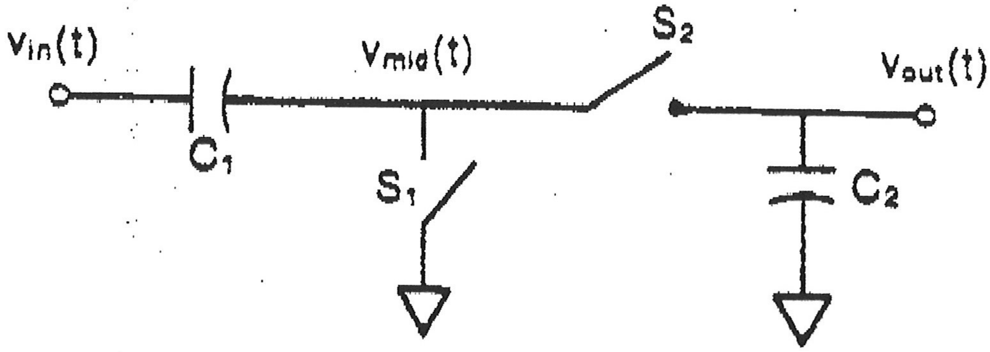
1. M. H. White, D. R. Lampe, F. C. Blaha and I. A. Mack, "Characterization of Surface Channel CCD Image Arrays at Low Light Levels," *IEEE J. Solid State Circ.* SC-9, 1, 1974.
2. R. J. Kansy, "Response of a Correlated Double Sampling Circuit to 1/*f* Noise," *IEEE J. Solid State Circ.* SC-15, 373, 1980.
3. G. R. Hopkinson and D. H. Lumb, "Noise Reduction Techniques for CCD Image Sensors," *J. Phys. E. Sci. Instrum.* 15, 1214, 1982.
4. R. W. Brodersen and S. P. Emmons, "The Measurement of Noise in Buried Channel Charge Coupled Devices," *Proc. Intl. Conf. Appl. CCDs*, Oct. 1975, San Diego.
5. H. M. Wey and W. Guggenbühl, "Noise Transfer Characteristics of a Correlated Double Sampling Circuit," *IEEE Trans. Circ. Sys.* CAS-33, 1028, 1986.

FIGURE CAPTIONS

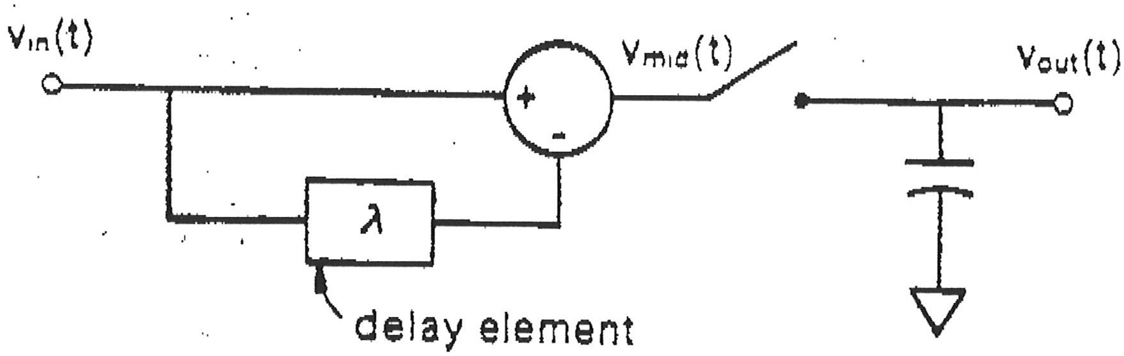
Figure 1 Two circuit implementations of correlated double sampling

Figure 2a Input and output power spectra for the correlated double sampling circuit with a high value of the system bandwidth

Figure 2b Input and output power spectra for the correlated double sampling circuit with a low value of the system bandwidth

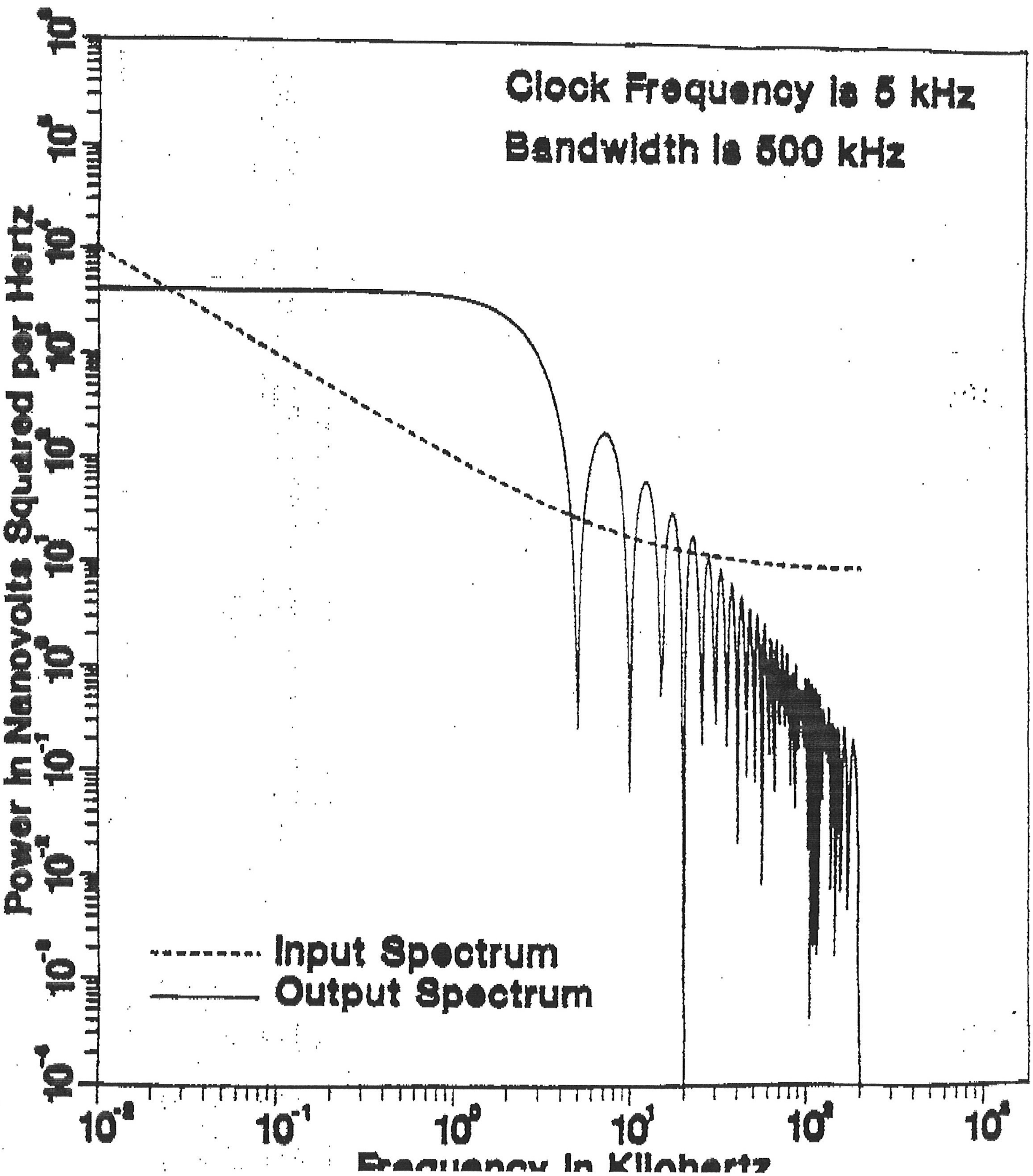


(a)



(b)

Input and Output Power Spectra



Input and Output Power Spectra