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Global Credit Research

July 1995

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Market Risk of the Step-Up Callable Structured Note

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Introduction

Structured notes are conventional debt instruments with embedded derivative transactions (generally forwards and options).[1] The derivative components of structured notes impact significantly the nature and magnitude of the market risk of the security.[2] Investors should therefore be thoroughly conversant with the derivatives lurking within the assets they plan to purchase. Failure to acquire this expertise may lead investors to pay excessive prices for structured notes and to expose themselves to unsuitable market risks.

One of the most prevalent structured notes is the "multi-step-up callable" for which the government-sponsored enterprises (GSEs) appear to be the primary issuers. This article describes this "step-up" callable and its elder cousins: the "plain vanilla" callable and puttable debt instruments. We find that a thorough understanding of these investments requires the ability to price the embedded derivative transactions. Armed with an approximate pricing methodology, we design and execute a *Monte Carlo* simulation to assess the market risk of these structured notes.

We conclude that the volatility of the market value of these callable and puttable notes is actually less than that of the underlying conventional notes. While not surprising in hindsight, a valuable lesson from this exercise is that embedded derivatives can decrease, as well as increase, risk. Moody's offers this analysis as a service to the investor community. Subsequent articles will focus on the risks of other structured notes and mortgage derivatives.

[1] See J.M. Pimbley and D.A. Curry, "Structured Notes and the Investor's Risk," *Moody's* Special Comment, March 1995.
[2] See J.M. Pimbley and D.A. Curry, "Credit and Market Risks of Corridor Notes/Swaps," *Moody's* Special Comment, September 1994.

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Description

Conventional callable and puttable debt instruments have proliferated for decades. In a callable note, the issuer has the right to redeem the note at a fixed price during a defined period prior to maturity. The issuer must declare this defined period and the redemption price upon sale of the securities.

Though not as common, puttable notes also have a long history. Such securities give the investor the right to sell the note back to the issuer at a fixed price during a defined period. Clearly, without any further thought, one may conclude that a puttable note must be more expensive than a similar instrument without the put provision since this latter feature must have value to the investor. By the same token, investors pay less for a callable note due to the call provision ceded to the issuer.

Let us now consider the multi-step-up callable note. The concept is a straightforward extension of the conventional callable. The underlying note has a variable, not fixed, coupon that increases with time. As in most callable notes, the step-up is callable at all coupon dates beyond the first call. This first call generally coincides with the first scheduled increase in the coupon. The issuer may prefer the step-up callable note over the conventional callable note since the initial (lowest) coupon of the former will be less than that of the latter.[3]

Pricing

The investor who purchases a callable bond risks having the bond called if interest rates fall sufficiently by a call date so that the underlying bond (without the call provision) would trade above the call price. The issuer is long the option and the investor is short. The call price is at par or above and the issuer may call the bond at any coupon date from the earliest call to maturity.

If the call price is at par, say, then the bond call option is equivalent to an option on a swap. As justification, consider that the issuer may buy the bond for par at a call date. Suppose instead that the issuer had an option to enter a swap to receive a fixed rate equal to the bond coupon and pay a LIBOR-plus-a-spread floating rate. If the issuer invokes the swap, the issuer will have converted the fixed-coupon bond to a floating-rate bond. If the spread in the swap is equal to the issuer's credit spread (measured with respect to the LIBOR/swap curve), this new floating-rate bond will trade at par. Hence, the option on the swap allows the issuer to convert the bond to a par-value instrument.

Since both the call option of the bond and the option on the swap produce the same result (a par-value investment), the values of the two options must be identical. Hence, we consider the call option problem to be a swaption problem.

Unfortunately, we've neglected the issue of the credit spread. That is, the issuer might choose to call the bond due to a narrowing of its credit spread even when the yield curve has not fallen significantly. We are not aware of any treatment of the call option that incorporates the credit spread. We shall ignore it as well. As we demonstrate in Appendix I, the value of the credit spread option is significantly less than that of the interest rate option when the issuer credit quality is high (eg., **Aa** or **Aaa**). The credit portion becomes increasingly relevant as the issuer debt rating declines.

The bond put option is analogous. The investor's long put option is equivalent to a long position in an option to enter a pay fixed, receive floating swap.

[3] While possibly valid in some circumstances, this intuitive explanation of why an issuer might prefer the step-up callable note to the conventional callable misses a key facet of the structured note market. Issuers usually "swap out" the derivatives embedded in their structured debt issues. With the callable or step-up callable notes, then, the issuer would likely sell the embedded swaption to a counterparty - often the dealer for the debt issue - at issuance. Hence, the choice of whether to issue the callable or step-up (or any other possibility) depends on what investors are willing to buy at the price most favorable to the issuer.

Like the callable note, the step-up callable note is simply an underlying note (with variable coupons, though) and a short (to the investor) option on a swap. The fixed-rate legs of this swap match the variable note coupons. Hence the requisite analysis for pricing and for market risk assessment is equivalent to that for the conventional callable and puttable notes.

Appendix II describes in detail our analytical approach to the embedded swaptions that comprise callable, puttable, and step-up callable structured notes. The most prominent conceptual assumptions are that the party long the option will invariably exercise when it is economically advantageous to do so and that the "Black framework" is valid. The model accepts a term structure of volatility and appears robust, consistent, and sensible.

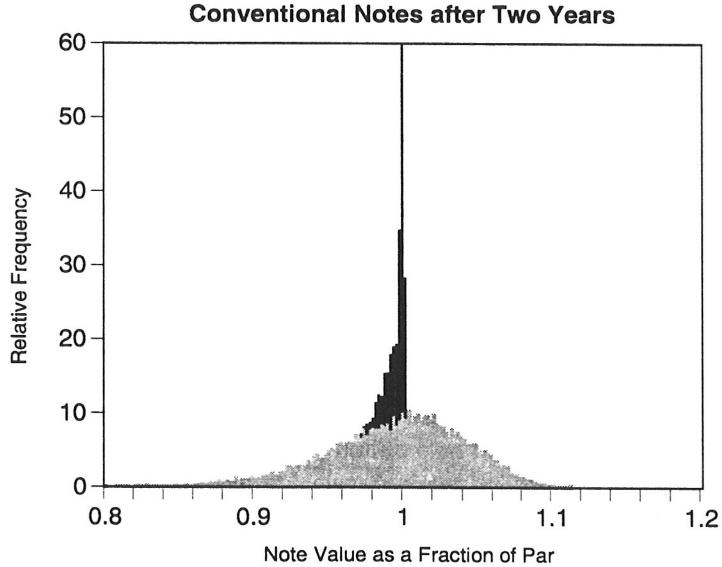
Market Risk

A simple and useful measure of market (interest rate) risk is the "effective duration" of a debt security. Adding either a call or put provision will generally reduce this duration measure, and hence the market risk, significantly. While a useful indicator, however, the duration is not a complete risk assessment tool.

To improve this risk measurement, we ask what will be the "likely values" of conventional, callable, and puttable notes at some point in the future. Clearly we can compile at best "probability density functions" for future values of these instruments. We generate these functions with a *Monte Carlo* simulation of the term structure of interest rates. This simulation allows the yield curve to vary stochastically (i.e., probabilistically) with imperfect correlation among different points on the curve and with an input term structure of volatility. At each point in time the algorithm reprices the note and all embedded swaptions. Each *Monte Carlo* trial generates a different note price. By running tens of thousands of such trials one maps out a plot (the probability density function) of the relative frequency of occurrence of each particular potential note value.

A comparison of a conventional and callable note appears below:[5]

Probability Density Functions for Callable and Conventional Notes after Two Years



This figure describes the range of values of the five-year notes three years prior to scheduled

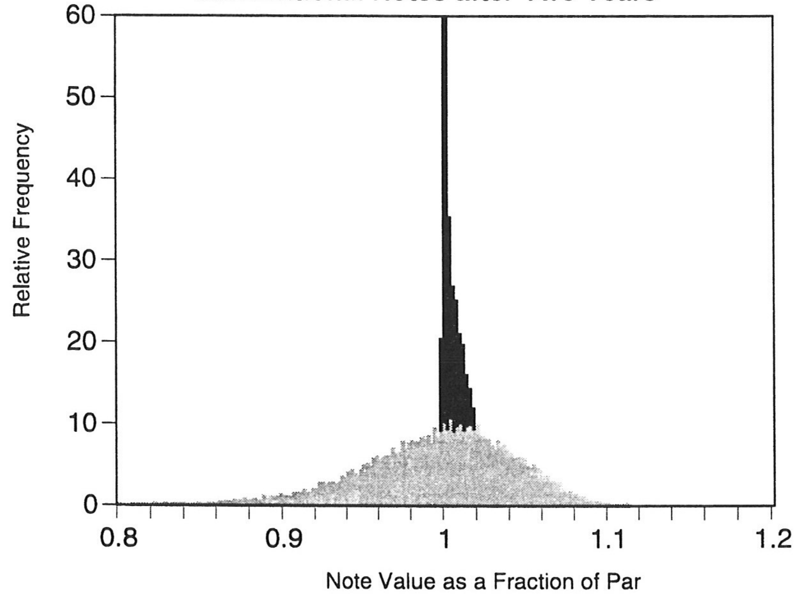
[4] The effective duration is the negative (calculus) derivative of the security price with respect to a parallel shift in the yield curve.

[5] The conventional note pays a semi-annual coupon of 7.28% while the callable note coupon is 8.24%. The issuer may call the latter after two years or at any coupon date thereafter at par. Both instruments have five-year maturities and have values near par with a **A** **a** issuer and the LIBOR/swap yield curve of late March 1995.

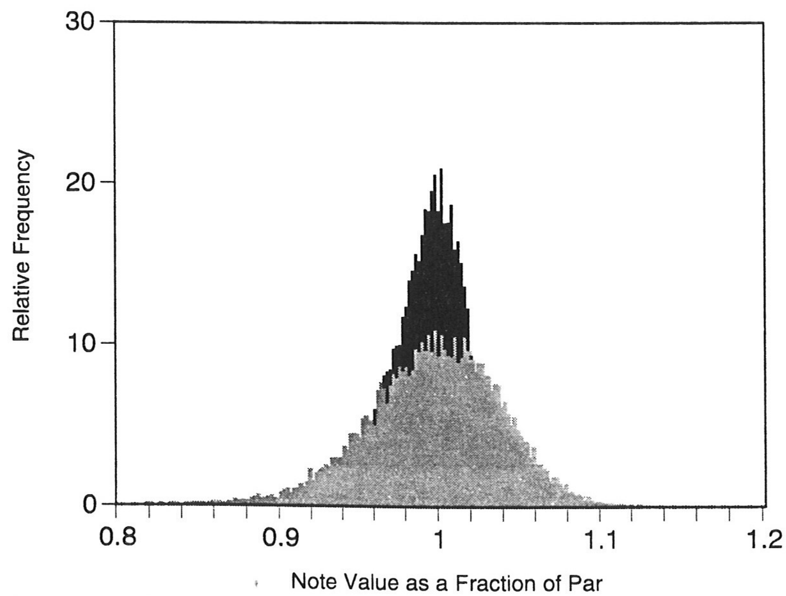
maturity (i.e., two years after issuance). The conventional note coupon is set at the value (7.28%) consistent with an initial value of par. We chose this two-year point as that moment in time immediately following the first call date of the callable note. In fact, a value of par for this callable note implies that the bond was indeed called at par. The callable note density function has the sharp peak at par and "zero probability" at note values greater than par while the conventional note has the gradual density function that is nearly symmetric about par.

Next we consider the puttable note:[6]

Probability Density Functions for Puttable and Conventional Notes after Two Years



Probability Density Functions for Callable and Conventional Notes after One Year



The situation of the first of these two graphs is reversed. The puttable note will have a value of par after two years only if the investor has just put the note at par. Otherwise, the market value of

[6] The conventional note pays a semi-annual coupon of 7.28% while the puttable note coupon is 5.81%. The investor may put the latter after two years or at any coupon date thereafter at par. Both instruments have five year maturities and have values near par with a **Aa** issuer and the LIBOR/swap yield curve of late March 1995.

this note will be above par.

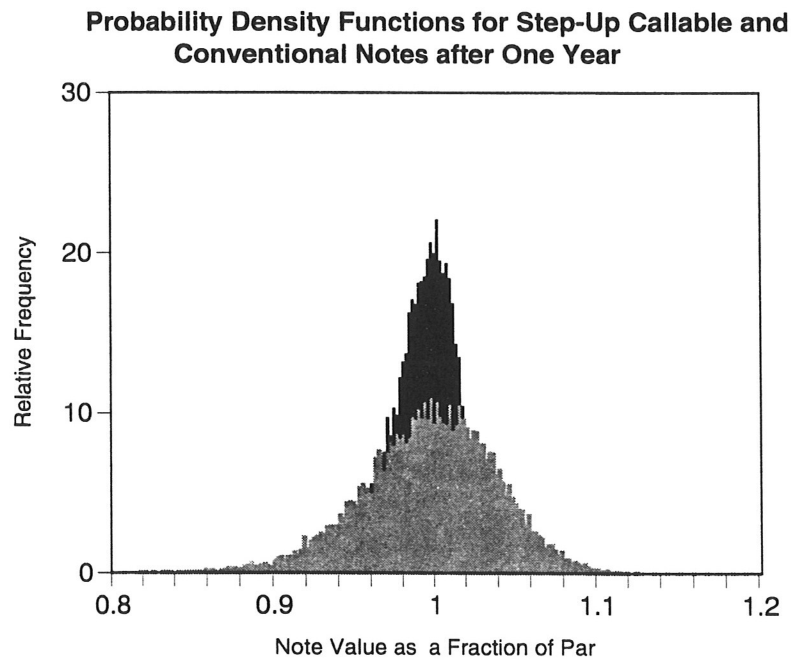
Clearly the abrupt cut-off feature of the callable and puttable note density functions results from our choice of the time (two years) coinciding with the first call/put exercise date. The impact of the call/put provision on the note price is still significant at times prior to the sequence of exercise dates as demonstrated by the second of the two preceding graphs.

The callable note probability density function is still narrower and more peaked than that for the conventional note. Further, though difficult to see in this reproduction, the callable note distribution remains asymmetric. That is, the likely values are not evenly distributed about par but rather are skewed to sub-par values. Both tendencies are less pronounced when compared with the earlier distributions after two years since there is significant "time value" remaining in the swaption at one year after issuance.

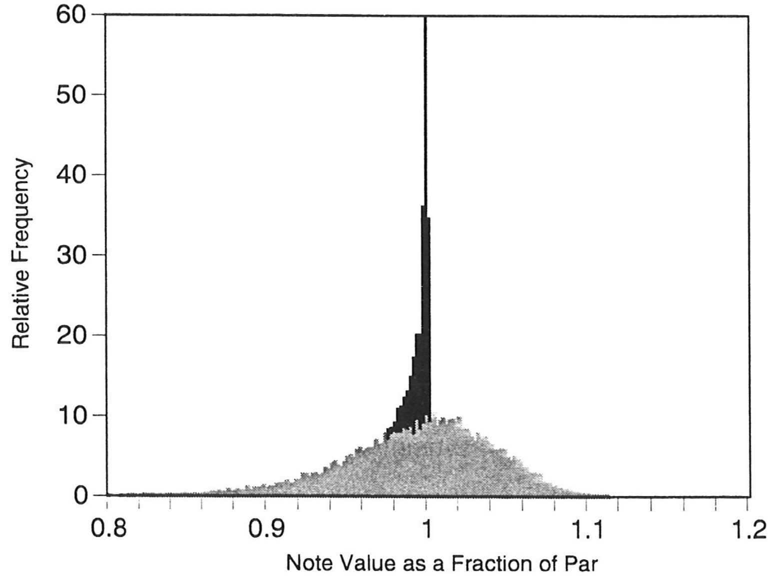
The Multi-Step-Up Callable Note

As a specific example of the step-up callable note, imagine a five-year note with semi-annual coupons with value near par "today". Consistent with the example in Appendix II, the issuer may call the note at par in two years or at any coupon date thereafter. Let the scheduled coupon be 8.1% for the first two years, 8.4 % for the third year, 8.8% for the fourth year, and 9.25% for the fifth year. Thus, the first call date matches the point at which the coupon steps up from 8.1 % to 8.4%. Again, this particular choice of coupon schedule implies an initial debt instrument value of par (with the market yield curve of March 1995). The issuer's beginning coupon of 8.1% is less than the 8.24% fixed coupon of the earlier plain vanilla callable note.

Below we attach plots of the probability density functions for this step-up callable note value after one and two years:



Probability Density Functions for Step-Up Callable and Conventional Notes after Two Years



Our earlier observations regarding the probability density function of the "plain vanilla" callable note apply here as well. The impact of the step-up feature is two-fold. First, the investor's downside is slightly diminished in the step-up callable (which is a reasonable benefit given the investor's lower coupon of 8.1%). Second, and quite related, is the observation that the issuer is more likely to call the step-up note after two years than it is to call the standard callable note. The issuer has more incentive to avoid paying the next 8.4 % coupon than to avoid the 8.24% coupon of the standard structure.

Market Risk Implications

The figures in this memorandum paint a clear picture. The variance in future values of callable and step-up callable notes is significantly less than that of corresponding conventional notes. (This statement stems from our association of the width of the security price probability density function with the variance of these prices.) By any sensible measure, then, a "volatility rating" or "market risk rating" on a step-up callable structured note will not indicate that this instrument has enhanced market risk.

One may certainly argue that we've considered a very narrow range of examples for such a sweeping observation. But the result is arguable in much simpler terms. An investor who buys a fixed-coupon note has interest rate risk. In selling the call provision to the issuer, the investor is short an instrument (the American put swaption) with the same (direction of) interest rate sensitivity as the underlying fixed-coupon note. Thus, the presence of the call provision reduces the investor's net long position on the yield curve. The same tautology reveals that puttable notes are less volatile than conventional, fixed-coupon notes as well.

As a parting shot, one often hears callable notes have "re-investment risk". That is, the issuer will opt to call a note when interest rates have fallen in order to replace its funding at lower cost. The issuer thereby compels the investor to accept his/her investment back at a time when interest rates for re-investment are low. Though technically correct, these statements do not imply, as many people suppose, that callable notes have "more" risk than conventional notes. Just the opposite is true. A callable note has smaller day-to-day fluctuations in market value than does its conventional counterpart.

Appendix I: Credit Spread Option Value

We noted earlier that we have not incorporated in our valuation of the bond call and put provisions the component due to varying credit spread of the issuer. That is, a bond value may rise above a strike level and be called, or fall below a strike level and be put to the issuer, due to fluctuations in the credit spread instead of interest rates. This appendix approximates the credit spread contribution for the purpose of gauging the inaccuracy in omitting this term in our earlier analysis.

As a crude exercise for estimating the issuer's credit spread option value in its call provision, we first take the forward credit spread equal to the initial value (an oversimplification). We then employ the standard Black put option valuation with an expiration of two years for comparison with the European swaption example of the memorandum. With a volatility of 40% that loosely reflects our experience with credit spreads, we find the option value to be 50 bp per 100 bp of original spread. That is, if the initial spread is 100 bp (typical of a **Baa** or **A** issuer), then the option value is 50 bp. If the initial spread is 20 bp (typical of a **Aaa** issuer), then the option value is 10 bp and so on. In this simple approach, the credit spread option embedded in the investor's put provision is identical (since the option is at-the-money with our assumption that the present and forward credit spreads are equal).

In Appendix II we find typical European swaption values to be 116 bp and 177 bp for the call and put provisions, respectively. Hence, the 10 bp spread option of a **Aaa** issuer has only a small, though not completely negligible, impact on the total swaption value. If the issuer were of speculative grade (spread of 300 bp or more), however, the credit component would equal or exceed that due to interest rate fluctuations.

The message is clearly that it appears reasonable to omit the credit spread contribution when the issuer is of high credit quality (e.g., **Aa** or **Aaa**). Such an omission becomes less and less tenable as credit quality declines. We expect that this statement holds when comparing the American credit spread option with the American interest rate swaption.

Appendix II: Pricing the Embedded Swaption

The European Swaption valuation is the first step

We shall refer to the interest rate swaption in question as "American" even though we must qualify this adjective with two comments. First, it might be more precise to use the term "Bermudan" to denote the feature that the option is exercisable at a fixed set of dates in the future and not at any arbitrary time. It appears to be accepted market practice to retain "American".

Second, both "American" and "Bermudan" are somewhat misleading in that the terms fail to convey the point that the option payout changes at each expiration date when the holder chooses not to exercise the option. That is, if the option holder exercises the swaption "today", he/she receives the value of the underlying swap. If the holder declines to exercise, the underlying swap of the swaption changes.

As a "warm-up exercise", let us consider the simpler case of a European swaption. Imagine a five-year note with semi-annual coupons that is callable in two years and at no time thereafter. The existence of only one call date is unusual. As discussed previously, the issuer is long an option on a swap to receive fixed and pay floating. With only a single call date, the swaption is European.

We adopt a "Black-type" valuation framework for the European swaption. The underlying swap extends from year two to year five (with the first payment exchange at two years and six months). Let F be the forward value at year two of all fixed swap payments. Let X be the forward value at year two of all floating swap payments. Further, z is the "zero coupon discount factor" for payments made at the option exercise date (at year two) and t is the time until expiration. The volatility of the sum of floating payments is σ.

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[7] Observe that the equivalent swaption for bond **call** option is a **put** swaption. That is, the issuer has the option to "sell" the swap (*i.e.,* receive the fixed rate in exchange for the floating rate) to the investor/writer. Conversely, the equivalent swaption for a bond **put** option is a

The option payout is ***max* { 0 , X — F }**. Given the similarity to equity and foreign exchange options and the plausibility of a log-normal process for the sum of floating payments, we argue that the option value is simply the Black expression:[7]

$$\textit{Swaption value} \;=\; z\,B_p\left(X, F, \sigma, t\right) \qquad (1)$$

where the "Black put function" B_p is

$$B_p \;=\; X\,\Phi\left(-d + \sigma\sqrt{t}\right) - F\,\Phi\left(-d\right) \qquad (2)$$

$$\textit{with} \quad d \equiv \left[\log(F/X) + \sigma^2 t/2\right] / \sigma\sqrt{t}$$

$$\textit{and} \quad \Phi(x) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} du\, e^{-u^2/2}$$

For the corresponding investor's bond put option with only one exercise date, the swaption payout is ***max* { 0 , F — X }**. Here the bond put option value is a similar Black expression:

$$\textit{Swaption value} \;=\; z\,B_c\left(X, F, \sigma, t\right) \qquad (3)$$

where the "Black call function" B_c is

$$B_c \;=\; F\,\Phi\left(d\right) - X\,\Phi\left(d - \sigma\sqrt{t}\right) \qquad (4)$$

The application of a Black-Scholes framework is somewhat crude in that the forward value of the sum of fixed-rate payments is not truly a constant value as is the strike of a conventional option. This sum of fixed-rate payments is not constant since the yield curve with which the future payments are discounted fluctuates. A consolation is that this fixed-rate payment term is significantly more stable than the floating-rate payment counterpart.

Further, the market legitimizes the Black-Scholes view of swaptions by quoting "swaption volatility" to express the value of a particular swaption. In fact, it would make sense for the swaption volatility σ to be less than, but nearly equal to, a typical volatility for a forward six-month LIBOR rate. The validity of this prediction qualitatively vindicates the Black-Scholes framework.

We shall briefly quote numerical results for the unusual call and put provisions of this section. Without such provisions, a **Aa** issuer would set the semi-annual coupon at or near 7.28 % to sell the five-year notes at par. The issuer's option to call the notes at par after two years (and at no time thereafter) has a value of 116 bp upon evaluating equations (1) and (2). To sell these callable notes at par, the issuer would raise the coupon to 7.67 %.

If instead the investor has the option to put the note at par after two years (and at no time thereafter), equations (3) and (4) place this option value at 177 bp. The investor should expect a reduced coupon of 6.64 % to compensate the issuer for the put option. The upward slope of the yield curve renders the put option more valuable than the call option.

The next level is the American swaption

To some extent the previous section was largely irrelevant because all issuer call and investor put provisions of which we are aware are of the "American" variety as opposed to "European". Still, the European framework provides an intuitive and analytical foundation upon which we shall build the valuation scheme for the American swaption.

[8] Numerical results of this memorandum employ the USD swap and Treasury yield curves of late March 1995.

When we say that the true call provision is "American", what we mean is that the issuer has the option at each call date except the last to call the bond at the pre-designated price or to wait until the next call date. That is, the issuer's decision is not simply to call now or **never** call (as in the European example of the earlier section).

This observation raises two points. First, the American swaption must be more valuable than its European counterpart since the latter invokes "all-or-nothing" while the former is "all-or-something". Second, for both the current valuation and subsequent *Monte Carlo* simulations, we must consider what will be the rational exercise criterion. For the European swaption, the issuer will exercise when the underlying swap has positive value. For the American swaption, the issuer may choose to accept the underlying swap or an American swaption of reduced maturity. The American swaption exercise decision, then, rests on the relative values of the swap and diminished maturity swaption. The issuer will exercise the swaption if and only if the swap has greater value than the diminished swaption.

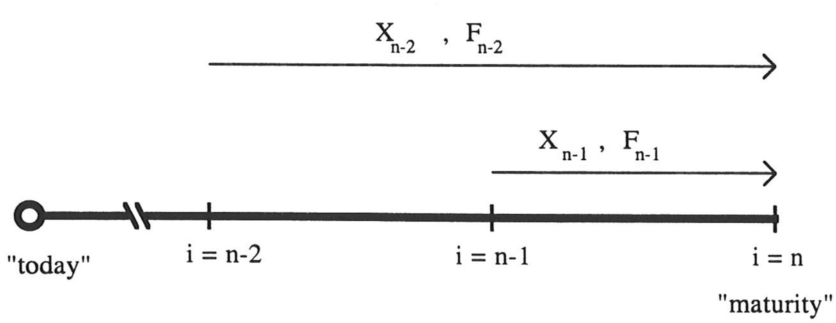
We develop "backward recursion" for the American swaption

This section develops a valuation technique for the American swaption. As a broad overview, we find it easiest to work backwards. That is, imagine we have a five-year note with semi-annual coupons that the issuer may call at year two or at any coupon date thereafter. We first find the value of the swaption with exercise date at 4.5 years after issuance (i.e., just six months prior to maturity). Given this 4.5 year swaption value, we find the value of the swaption exercisable at 4 years. Then we get the swaption value at 3.5 years and so on until we reach the true (first) exercise date at 2 years.

Why do we proceed in this manner? Though it may not be obvious, it greatly simplifies the problem. The value of the American swaption at any point in time depends on the values of the "subordinate" swaptions associated with subsequent call dates. Thus, it is sensible to compute these "subsequent call date swaptions" first.

The derivation that follows harbors an approximation above and beyond that of the Black-Scholes construct. One may defend the approximation qualitatively on the grounds that we substitute a mean value term within an integrand and that the subsequent integration reduces the error due to cancellation of over-compensating and under-compensating regimes. But qualitative assurances are not sufficient. We provide a more detailed error discussion at the point we introduce the approximation.

Consider the illustrative sketch below which may help explain our notation:



The points in time labelled *"i = n - 2", "i = n - 1",* and *"i = n"* denote the final three coupon payment dates with the last date also coinciding with bond maturity. The symbols X_{n-1} and F_{n-1} stand

[9] We choose to explicity write only the time-dependence as an argument of Π_{n-1} merely to simplify equations that follow. It is understood that this swaption value varies with volatility and fixed and floating payments as well.

for the fixed and (expected) floating payments, respectively, in the final coupon period of the bond. Likewise, X_{n-2} and F_{n-2} signify the sums of expected fixed and floating payments over the final two coupon periods.

Our "backward marching" (or "backward recursion") method begins by valuing the last swaption with exercise time at $i = n - 1$. This swaption is effectively European since there are no "trailing" swaptions. Hence the value of the (put) swaption for a bond call option — which we call $\Pi_{n-1}(t_{n-1})$ — replicates the European swaption expression of equation (1):[9]

$$\Pi_{n-1}(t_{n-1}) = z_{n-1} B_p(X_{n-1}, F_{n-1}, \sigma, t_{n-1}) \qquad (5)$$

where t_{n-1} and z_{n-1} are, respectively, the time to the call date (at $i = n - 1$) and the zero coupon discount factor for payments made at this call date. Similarly, the value of a (call) swaption for a bond put option, $\Gamma_{n-1}(t_{n-1})$ replicates equation (3):

$$\Gamma_{n-1}(t_{n-1}) = z_{n-1} B_c(X_{n-1}, F_{n-1}, \sigma, t_{n-1}) \qquad (6)$$

We now move to the penultimate swaption values: $\Pi_{n-2}(t_{n-2})$ and $\Gamma_{n-2}(t_{n-2})$. At exercise *(i.e.,* when t_{n-2} is zero), the swaption payouts are:

$$\Pi_{n-2}(0) = \max\left[X_{n-2} - F_{n-2}\,,\ \Pi_{n-1}(\Delta)\right] \text{and}$$

$$\Gamma_{n-2}(0) = \max\left[F_{n-2} - X_{n-2}\,,\ \Gamma_{n-1}(\Delta)\right]$$

The Δ in these equations is the time period between coupon payments (*i.e.,* one-half year in our current example). At exercise, the option holder will enter the underlying swap if the first of the two bracketed terms exceeds the second. If instead the second term exceeds the first, then the subordinate swaption has more value than the current swap and the holder's rational decision will be to defer exercise of the swaption.

The greater challenge lies in deriving this penultimate swaption value prior to expiration. As is typical of option pricing endeavors, we write the (put) swaption value for the bond call provision as the present value of the swaption payout weighted with the risk-neutral probability density function (pdf) of the forward floating rates:

$$\Pi_{n-2}(t_{n-2}) =$$
$$z_{n-2}\int_0^\infty d\eta_{n-1}\int_0^\infty d\eta_{n-2}\, p(\eta_{n-2},\eta_{n-1};F_{n-2},F_{n-1})\max\left[X_{n-2}-\eta_{n-2}\,,\ \Pi_{n-1}(\Delta)\right]$$

By writing

$$\max\left[X_{n-2}-\eta_{n-2}\,,\ \Pi_{n-1}(\Delta)\right] = \Pi_{n-1}(\Delta) + \max\left[X_{n-2}-\eta_{n-2} - \Pi_{n-1}(\Delta),\ 0\right]$$

we find

$$\Pi_{n-2}(t_{n-2}) = \Pi_{n-1}(t_{n-1}) +$$
$$z_{n-2}\int_0^\infty d\eta_{n-1}\int_0^\infty d\eta_{n-2}\, p(\eta_{n-2},\eta_{n-1};F_{n-2},F_{n-1})\max\left[X_{n-2}-\eta_{n-2} - \Pi_{n-1}(\Delta),\ 0\right]$$

We now reach the point of our analytical approximation. The $\Pi_{n-1}(\Delta)$ within the integrand is a function of the floating rate variable of integration η_{n-1}. Yet it provides a tremendous simplification, to say nothing of an elegant result, to make the replacement.

$$\Pi_{n-1}(\Delta) \Rightarrow \Pi_{n-1}(t_{n-1}) / z_{n-2}$$

The term on the right-hand side above is just "today's" value of the swaption to be exercised at $i = n - 1$ divided by the zero coupon discount factor. Hence, this right-hand side is the forward value of the put swaption. The substitution is essentially that of replacing a variable within the integrand by a mean value.

What effect does this substitution have on the eventual accuracy of the swaption value? When the swaption is far in-the-money, the approximation should be exceedingly accurate since the integrals of the two quantities

$$\Pi_{n-1}(\Delta) \text{ and } \Pi_{n-1}(t_{n-1}) / z_{n-2}$$

multiplied by $p(\eta_{n-2}, \eta_{n-1}; F_{n-2}, F_{n-1})$ are equal. Also, the case in which the swaption is far out-of-the-money presents no difficulty since the integral contribution to the swaption value, even if inaccurate, will be much less than the contribution of the subordinate swaption value (i.e., the additive term). The remaining case is that of the swaption at-the-money. Here the approximation will overestimate the integral term by an amount that is difficult to quantify without performing direct comparisons with presumably accurate numerical simulations. As an estimate, it appears the error could approach or exceed ten percent of the true swaption value. A comforting feature is that the backward recursion technique we develop here will tend to damp error growth. That is, an overestimate of the swaption value at one step will depress the value of the swaption with the next earlier exercise date.

With this approximate substitution, we get

$$\Pi_{n-2}(t_{n-2}) = \Pi_{n-1}(t_{n-1}) + B_p\left[\tilde{X}_{n-2} - \Pi_{n-1}(t_{n-1}),\ \tilde{F}_{n-2},\ t_{n-2}\right] \qquad (7)$$

where $\tilde{X}_i = z_i X_i$ and $\tilde{F}_i = z_i F_i$. This is great! The penultimate swaption value is a simple function of the subordinate swaption value and has the interpretation as just a modified European swaption. The progression for valuing the swaptions expiring at $i = n - 3$, $i = n - 4$ and so on is trivial. Given the subordinate swaption at i, the value of the swaption at $i - 1$ is just

$$\Pi_{i-1}(t_{i-1}) = \Pi_i(t_i) + B_p\left[\tilde{X}_{i-1} - \Pi_i(t_i),\ \tilde{F}_{i-1},\ t_{i-1}\right] \qquad (8)$$

The corresponding expression for the (call) swaption pertaining to the bond put provision is

$$\Gamma_{i-1}(t_{i-1}) = \Gamma_i(t_i) + B_c\left[\tilde{X}_{i-1} + \Gamma_i(t_i),\ \tilde{F}_{i-1},\ t_{i-1}\right] \qquad (9)$$

The call/put option algorithms have appealing properties

Equations (5) and (8) describe completely the technique for valuing the American put swaption while equations (6) and (9) serve the same purpose for the call swaption. The key idea is that one first values the last piece of the swaption as European and then progressively adds new components moving backward until one reaches the true first exercise date.

This construction has several distinct advantages. First, it is easy to implement. Though it may seem unwieldy in its recursive form, bear in mind that such pricing formulae are always evaluated in computer codes and spreadsheet programs. Hence, this American swaption valuation is only slightly more cumbersome than European swaption valuation.

Second, the form of the recursion (equations (8) and (9)) clearly shows, for example, that put or call American swaptions with first exercise at two years are more valuable than those with first exercise at any time greater than two years. This property is fairly obvious without the mathematics, but it is nonetheless heartening to see it replicated so painlessly by our approximate valuation. Further, it is clear that if the nearest swap is well out-of-the-money so that swaption exercise at the upcoming call/put date is highly unlikely, the swaption valuation essentially reduces to that of the subordinate swaption.

Third, the recursive expressions easily admit the possibility that the bond call/put provision may require a premium. That is, the call price may be above par or the put price below par. For a call price premium ξ_i , we simply add this amount (discounted to present value) to the floating rate "payment". We make a similar adjustment for a put price premium ς_i . Equations (8) and (9) become:

$$\Pi_{i-1}\left(t_{i-1}\right) = \Pi_i\left(t_i\right) + B_p\left[\tilde{X}_{i-1} - \Pi_i\left(t_i\right),\ \tilde{F}_{i-1} + \xi_{i-1},\ t_{i-1}\right] \tag{8a}$$

$$\Gamma_{i-1}\left(t_{i-1}\right) = \Gamma_i\left(t_i\right) + B_c\left[\tilde{X}_{i-1} + \Gamma_i\left(t_i\right) + \varsigma_i,\ \tilde{F}_{i-1},\ t_{i-1}\right] \tag{9a}$$

Finally, the formulation lends itself well to flexibility in the input specifications. That is, there is no additional labor required to impose a term structure of (swaption) volatility or to vary the call/put premium and strike level at each exercise date.

Preliminary Results

As a preliminary demonstration of the results of this analysis, let us consider again the call and put provisions of the earlier section on European swaptions. We imagine a **Aa** issuer that would set the semi-annual coupon on five-year notes at or near 7.28 % to sell the debt issue at par. The issuer's option to call the notes at par after two years (or at any coupon date thereafter) has a value of 243 bp upon evaluation of the recursion of equation (8). To sell these callable notes at par, the issuer would raise the coupon to 8.24 %. (The comparable European swaption values were 116 bp and 7.67 %.)

If instead the investor has the option to put the note at par after two years (or at any coupon date thereafter), the recursion of equation (9) places this option value at 377 bp. The investor should expect a reduced coupon of 5.81 % to compensate the issuer for the put option. The upward slope of the yield curve again renders the put option more valuable than the call option. (The comparable European swaption values were 177 bp and 6.64 %.)

These examples produce drastic changes in the coupon level due to the high value of the call/put provisions. More typically, such bond call and put options would specify premiums that would reduce the option values and hence produce less dramatic changes in coupons.

The call/put provisions were more than twice as valuable in the "American form" than in the "European form". The inequality is sensible from a qualitative standpoint since the party long the option has much more flexibility (i.e., option value) in the American exercise mode. The two-to-one ratio in option value will change with any modification in the terms and conditions of the swaption.