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Credit and Market Risks of Corridor Notes/Swaps

New exotic derivatives magnify profit and loss and leave some investors ignorant of their increased downside risk. Corporate issuers and financial firms have recently created a new class of derivative instruments to provide investors with high yields and commensurately high risks. The "corridor note" (or "range note/bond"), a specific example of a "structured note", and "accrual range floater" (or "corridor") swap are similar in form to conventional notes and swaps. But the investor receives an above-market note coupon or generous swap terms in exchange for agreeing to forgo coupon or swap payments in the event that LIBOR (the London InterBank Offer Rate) falls outside prescribed bounds (the "corridor").

Pricing these products is challenging. The potential exists for the investor to seriously underestimate the "corridor risk" in evaluating both the note and swap variants.* The most widely publicised losses are those of the BankAmerica Pacific Horizon Prime fund ($67.9M capital injection) and the Piper Jaffray *Institutional Government Income Portfolio* (reported 23% fall in the $3.5B fund value). The bulk of the losses in the latter stemmed from mortgage derivatives. In this *Comment* we examine the risk of corridor notes and swaps in detail with the aid of analytical pricing methods and *Monte Carlo* simulations.

Though we do flesh out some credit risk issues, our emphasis is primarily market risk. *Moody's* offers this analysis as a service to the investor community. Subsequent articles will focus on the risks of other structured notes and mortgage derivatives. The complex nature of these instruments in concert with an investment grade credit rating of the issuer (e.g., **Aaa** government agencies) obscure the extent of the investors' risk. We wish to banish this obscurity.

Description

- The corridor note coupon varies with daily LIBOR settings
- The corridor-enhanced FRN coupon has more LIBOR sensitivity
- A range floater swap netted with a standard note produces a corridor-enhanced FRN
- A range floater swap netted with a floating rate note produces a corridor note
- The corridor note is similar to binary LIBOR municipal bonds
- Other notes and swaps have variable corridor ranges

The corridor note coupon varies with daily LIBOR settings

A typical corridor note might have a two year maturity and pay an 8.5% coupon semi-annually. (For simplicity, we shall consider the issuer to be highly rated so that its yield curve is equivalent to the LIBOR/swap yield curve. The specific numerical values we employ throughout this report are appropriate for the USD interest rate environment of early August 1994.) The market coupon for a conventional note is more than two hundred basis points lower at 6.4%. But the investor receives the high coupon only when (six month) LIBOR is in the range 4.5% to 7.5%. (Note that "today's" LIBOR is 5.25%.) The true coupon is computed on a daily accrual basis. Thus, if LIBOR falls inside the range for half of the days of the semi-annual coupon period and outside for the other half, the investor receives only half the 8.5% coupon. *Mitsubishi Finance* recently issued a corridor note of this type.

* The Securities and Exchange Commission (in a 30 June 1994 letter from the Director of the Division of Investment Management to the General Counsel of the Investment Company Institute), Office of the Comptroller of the Currency (in "Purchases of Structured Notes," OCC AL 94-2, 21 July 1994), and the Federal Reserve (in "Supervisory Policies Relating to Structured Notes," SR 94-45, 5 August 1994) have issued advisory warnings to banks on the risks of structured notes such as corridor notes. See the Swaps Monitor issues of 4 July, 18 July, 1 August, and 15 August 1994 as well as the 11 July and 18 July 1994 issues of Derivatives Week.

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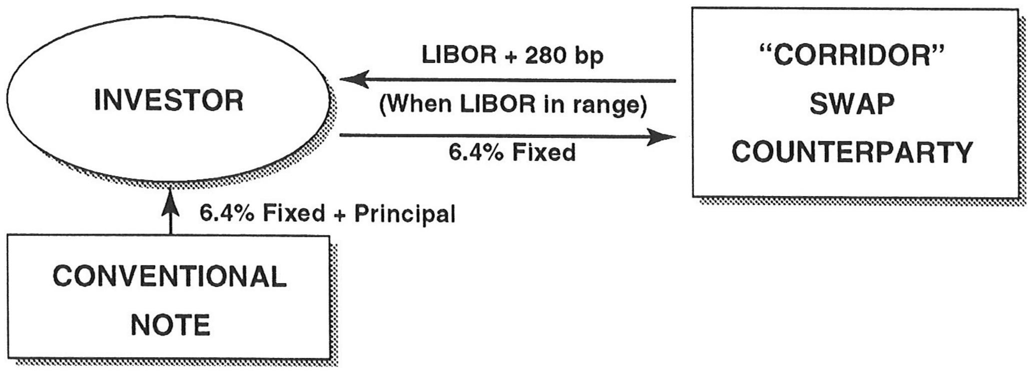
The corridor-enhanced FRN coupon has more LIBOR sensitivity

Or consider a "corridor-enhanced" floating rate note (FRN) of the same maturity in which the four coupon payments are LIBOR plus 280 basis points when LIBOR is between 4.5% and 7.5% and zero otherwise (computed, as before, on a daily accrual basis). The 280 basis point spread here and the 210 basis point spread (8.5% minus 6.4%) of the previous example arise from our analysis of the corridor note value we discuss in the next section. Both notes should trade at par with these spreads.

A range floater swap and a note produce a corridor-enhanced FRN

An investor may generate his/her own synthetic corridor-enhanced FRN by adding a type of "accrual range floater" swap to a conventional, fixed coupon note. The investor would pay fixed (6.4%) in this two year, at-the-money swap and receive LIBOR plus 280 basis points when LIBOR is within the corridor range of 4.5% to 7.5%. The swap payment to the investor, like the coupon payment of the associated note, follows a daily accrual calculation. See the diagram below:



A range floater and standard FRN make a corridor note

By the same token, the investor may synthesise the corridor note by pairing a conventional FRN (which pays LIBOR for each coupon) with an offer-side accrual range floater in which the investor pays LIBOR and receives 8.5% when LIBOR is inside the corridor and zero when LIBOR is outside. We shall base the remainder of this comment on these four cases (two notes and two swaps).

The corridor note is similar to binary LIBOR municipal bonds

The impact of the LIBOR setting on a note/bond coupon is also evident, though much more pronounced, in the binary LIBOR municipal bond ("*Goldman* Embeds Binary Structure in New York Muni Deal," *Derivatives Week,* 20 December 1993 and "Street Adds Equity Derivatives, Accrual Notes to Muni Market," *Derivatives Week,* 10 January 1994). This binary bond coupon is quite generous for an initial period but falls to zero for the remainder of the bond tenor if LIBOR on a specific date exceeds the strike level. The prescription of all future coupons based on a single day's LIBOR setting distinguishes the binary bond from the corridor note. The analysis of the former is more straightforward than that of the latter.

Other notes and swaps have variable corridor ranges

There also exist notes and swaps of this sort in which the corridor range is not constant. When the note maturity or swap tenor exceeds two years, it actually makes more sense to specify a corridor that varies with time due to the upward slope of the yield curve (and hence forward rates). Of greater interest is a corridor that varies with LIBOR. *PEMEX* (*Petroleos Mexicanos*) recently issued a three year note in which a "floating corridor" spans the range from LIBOR minus 25 basis points to LIBOR plus 50 basis points where the LIBOR-dependent corridor is set at each coupon payment.

Pricing

- Standard *Monte Carlo* pricing is insufficient for our study
- The corridor note contains daily digital caps and floors
- Investors underestimate the corridor risk
- The corridor-enhanced FRN also has standard caps and floors

Standard Monte Carlo pricing is insufficient for our study

A *Monte Carlo* simulation would provide the most expedient pricing method for these sorts of transactions due to their complexity. The financial engineer's custom software would permit the yield curve to vary in a risk-neutral manner over the two year tenor, record all coupon payments based on the LIBOR history, and repeat the process for tens of thousands of *Monte Carlo* trials to get expected values with high confidence.* But this methodology is computationally too expensive for our analysis. We wish not only to price the note or swap today but also to examine the *probability density function* for note and swap values at future times. Construction of this density function requires its own Monte Carlo simulation. The "nesting" of multiple levels of *Monte Carlo* simulation is prohibitive. Hence, we must work to define the mathematical expressions relevant to pricing these note and swap variants given all data including time to maturity, current yield curve and past history of LIBOR to eliminate one level of stochastic simulation.

The corridor note contains daily digital caps and floors

Of the two notes and two swaps described in the previous section, it is sufficient to look simply at the swaps. The two corridor notes are simply conventional notes, which we already know how to price, added to the swaps. Let us consider in particular the offer-side swap in which the investor pays LIBOR and receives a fixed rate accrued at 8.5% for LIBOR in the corridor and accrued at zero when LIBOR is outside.

Furthermore, we momentarily simplify the problem by ignoring the accrual feature. In this hypothetical case, the investor receives precisely 8.5% for LIBOR (measured only at the beginning of the semi-annual period) inside the corridor and precisely zero for LIBOR outside. We re-write this description of what the investor receives as follows:

Investor receives 8.5% (all LIBOR settings)
Investor pays 8.5% if LIBOR > 7.5%
Investor pays 8.5% if LIBOR < 4.5%

The reader may easily verify that the net result of these three statements is what we had previously pronounced: that the investor receives 8.5% if and only if LIBOR is in the corridor (from 4.5% to 7.5%).

But the three-statement version is actually more useful. One recognises the payments of the second and third lines as those of a digital cap and digital floor, respectively, with payout 8.5%.** The investor is short the options. Thus, our simulation need only price correctly these digital caps and floors. This requirement is fairly standard in the industry. One analyses the yield curve to get the appropriate forward rate (which doubles as the risk-neutral expected value of LIBOR) and inserts a volatility value into a digital option valuation expression. Caps are calls, of course, on the value of LIBOR while floors are puts. The appendix lists the appropriate mathematical formulae for conventional and digital caps and floors.

As we stated above, this formulation gives great weight to the LIBOR setting at the beginning of the (semi-annual) payment period. The payout of each cap and floor is either zero or 8.5% for the entire half year period. We must now relax the simplification in which we omitted the accrual aspect of the payment determination. Instead of specifying only one cap and floor for each period, we have in fact a cap and floor *for each day.* Every digital option within a payment period pays its contribution at the end of the period. Our simulations do not literally specify a cap and floor for every day. Rather, we space ten to twenty cap/floor combinations within each period.

Investors underestimate the corridor risk

Before we continue, we note that this is the point at which investors generally underestimate the "corridor risk". It is quite possible that the forward curve (of risk-neutral expected LIBOR values) lies entirely within the corridor over the swap tenor. Thus, the total **intrinsic value** of the digital options the investor has sold is zero. The investor may therefore have the erroneous impression that he/she is likely to receive the full coupon for the life of the swap. In fact, though, the **time value** of the digital options is quite high. And it is precisely this component of the option value that is most difficult to estimate.***

The corridor-enhanced FRN also has standard caps and floors

Let us now consider the other swap variant. The investor pays fixed (the market two-year swap rate of 6.4%) and receives LIBOR plus the 280 basis point spread when LIBOR is within the corridor. Again,

* The "risk-neutral" evolution is appropriate for pricing such derivative products. The future expected value of the underlying market variable is today's forward value.

** A "digital" or "binary" option pays a fixed amount upon exercise that is independent of the underlying asset value or interest rate as long as the "exercise condition" has been met. For example, a digital equity put option pays a fixed amount when the equity value finishes lower than the strike.

*** The "intrinsic value" of an option is simply the option payout formula evaluated with the present forward asset value or interest rate. The "time value" is the remainder of the total option value.

the investor receives nothing on a daily accrual basis when LIBOR is outside the corridor. As before, we seek to re-write the payment the investor receives:

Investor receives LIBOR + 2.8% (all LIBOR settings)
Investor pays LIBOR - 7.5% if LIBOR > 7.5%
Investor pays 7.5% + 2.8% if LIBOR > 7.5%
Investor receives 4.5% - LIBOR if LIBOR < 4.5%
Investor pays 4.5% +2.8% if LIBOR < 4.5%

The complexity of this case is greater than that of its predecessor. In order to "capture" the payout of LIBOR plus 280 basis points in terms of standard caps and floors, we must employ two caps and two floors. The conventional cap pays LIBOR minus *strike.* The digital cap pays whatever fixed amount we specify. To get our desired payout, then, we must set this digital fixed amount to *strike* plus 280 basis points. The same comments pertain to the floor. Careful consideration reveals that the investor is actually long the conventional floor. Thus, we now have four options: a short conventional cap, a short digital cap, a long conventional floor and a short digital floor.

Our simulation then would ideally specify a conventional swap loaded with four daily options - which we again approximate as ten to twenty sets of options per payment period. Pricing the individual options presents no difficulty. We merely aggregate them and combine payments at the end of each period.

We mentioned earlier that we shall not explicitly consider variable corridors (either pre-determined or tied to LIBOR). Let us comment at this point that a variable but pre-determined corridor range requires only sensible adjustments to option strike levels and (digital) payouts. When the corridor limits are tied to LIBOR, the caps and floors acquire a "resettable" or "forward-start" nature that one may easily incorporate into the mathematical valuations.

Credit and Market Risks

- Credit risk is the expected loss of the corridor derivatives
- The pdf (probability density function) provides a snapshot of market risk
- Our simulations give pdf's of accrual range floater swaps
- The range floater has significant probability of severe losses
- The corridor note pdf's are similar to those of the swaps

Credit risk is the expected loss of the corridor derivatives

The credit risk of a conventional note or bond measures the expected loss the investor will suffer due to incomplete principal and/or coupon payments. A corridor note or corridor-enhanced FRN simply carries the issuer's credit rating. One requires a custom credit risk analysis primarily for the associated (accrual range floater) swaps. A mutual fund, for example, that synthesises its own corridor note with a conventional note and the corresponding swap must compute the composite credit risk as the sum of expected losses on the two components.* Likewise the investment bank that transacts the swap with the fund must measure its credit exposure to the fund.

The pdf provides a snapshot of market risk

Market risk is relevant for both the notes and swaps. Our approach will be to calculate the probability density function for (which is really just a histogram of) note and swap values at an intermediate time between issuance and maturity. Since our examples have two year tenors, we examine the distribution of note and swap values immediately after the coupon and swap payments at twelve months. This point in time gives roughly the greatest variance in values emerging from the competing factors of time to maturity and extent of yield curve movements.

The previous section disclosed our ability to value these swaps and notes. The more arduous task of creating the probability density function requires that we simulate the yield curve fluctuations over time and value the swaps and notes at each point along the way. For the sake of clarity, we do not consider the note issuer credit spread as a separate variable in this analysis. Inclusion of the credit spread is not difficult, but it would obscure our comparative analysis. The credit spread is common to conventional and corridor notes.

* The "expected loss" is the sum of credit losses in all conceivable scenarios (e.g., note default, swap default, joint default of both the note and swap, et cetera) weighted with the probability of each scenario.

Our simulations give pdf's of accrual range floater swaps

We begin with "today's" LIBOR/swap yield curve. Our program takes adjustable time steps (generally of about one week) and varies each point on the yield curve with historical volatilities and correlations. The changes in yield values (apart from drift implied by forward rates) are taken to be log-normal. We continue a repetitive process by which we take steps in time, adjust the entire yield curve and re-value the swaps and notes with the embedded caps and floors. We continue the time steps until we reach the end of our simulation time (which in this case would be just more than a year). A simulation over the full two years would be uninspiring since the note price would finish at par (in the absence of default) and the swap value would return to zero.

Figure 1 **Distributions of Two Year Swap Values after One Year**

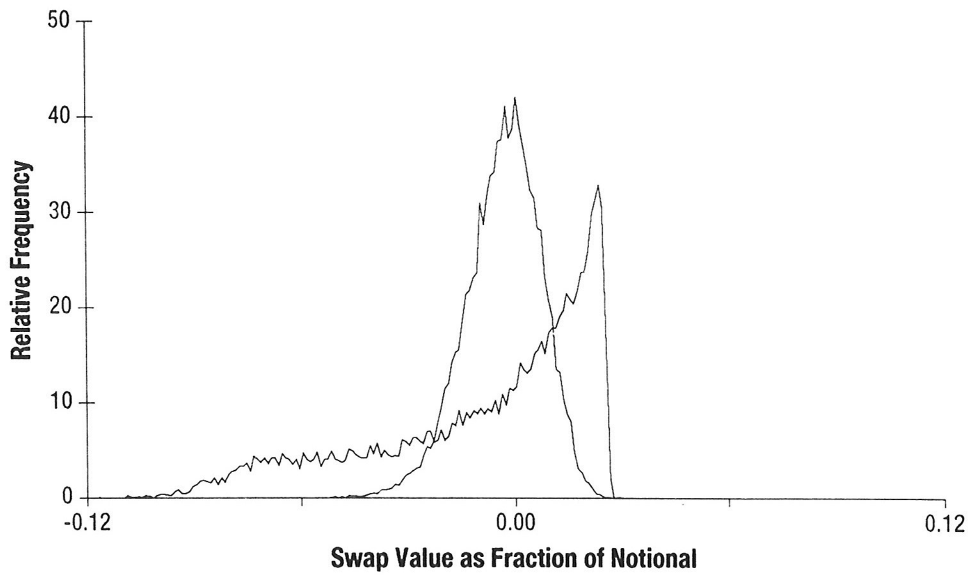


Figure 1 plots the probability density functions for two offer-side swaps resulting from fifteen thousand *Monte Carlo* trials. The curve that is nearly symmetric and has its peak near zero swap value corresponds to a standard swap in which the investor pays LIBOR semi-annually and receives the market fixed rate of 6.4%. The second curve, with peak position to the right of the standard swap, describes the accrual range floater swap in which the investor pays LIBOR and receives 8.5% fixed or zero depending on whether or not LIBOR is within its corridor.

The density function for the standard swap coincides nicely with one's intuition. The distribution is very slightly skewed to negative values as one expects in an offer-side swap based on an upward sloping yield curve. While there exists no law of economics requiring the swap value distribution to be symmetric, the rough symmetry of the standard swap is certainly plausible since there is no reason to suspect a marked asymmetry. Clearly the portion of the density function with swap value greater than zero corresponds to *Monte Carlo*-generated market environments with low LIBOR levels while the portion with swap value less than zero corresponds to high LIBOR levels.

The range floater has significant probability of severe losses

But the accrual range floater swap is a different story entirely. The peak at approximately two percent of the swap notional represents the investor's position if LIBOR essentially **does** remain well within its corridor over the course of the first year. But this upside cuts off abruptly above two percent.* Both positive and negative deviations of the yield curve tend to decrease the swap value and hence produce the extended downside tail on the distribution. Negative swap values arise both from LIBOR moving outside the corridor and from increased option values as LIBOR simply approaches the corridor boundaries.

These probability density functions make the point clearly. The investor has a greater upside potential in the accrual range floater relative to the standard swap but also a greater exposure to loss. In a

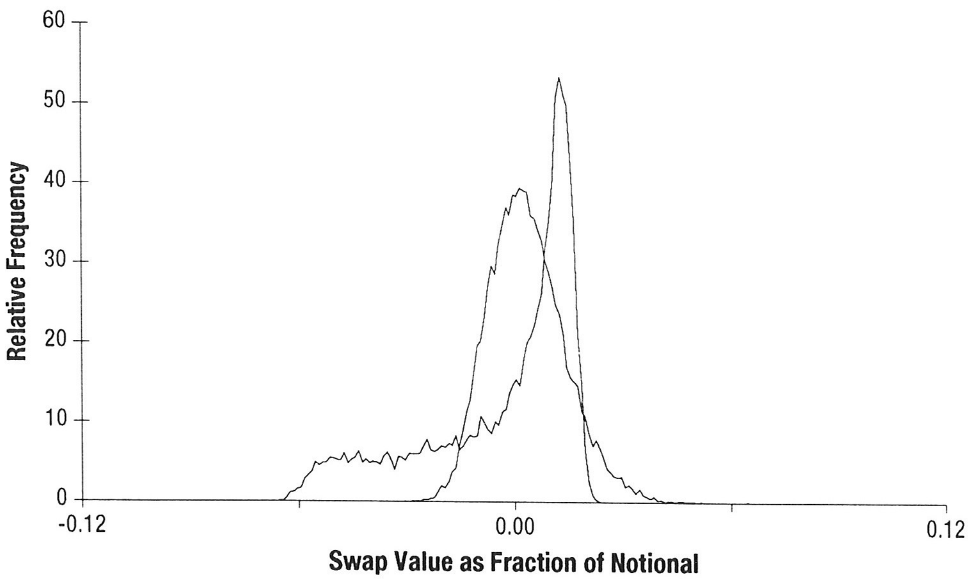
* The sharp cut-off is eminently reasonable. From the investor's point of view, the best scenario is that LIBOR is within the bottom half of the corridor range. If we could ignore the time value of the short floors and caps, we see that the investor will receive two 4 % payments in the final two semi-annual cash flows when LIBOR is near the 4.5 % lower band of the corridor. That is, the investor pays 4.5 % and receives 8.5 % (both multiplied by one-half since the payment period is half a year). If LIBOR falls below 4.5 %, then the investor will not receive the full 8.5 %. Thus, ignoring the time value of floors and caps, the swap cannot possibly be worth more than 4 % of notional with one year remaining. Factoring in the value of the short floors and caps, it is certainly plausible that the distribution of values for this range floater swap should cut off at about 3 % of notional.

$ 100 m swap, the investor has significant probability (about 14%) of experiencing a loss after one year of more than $ 6 m. The probability of a loss of this magnitude in the associated standard swap, on the other hand, is essentially zero (less than 0.01%).

Or consider the investment bank that has engaged the investor in this swap. The bank has credit risk when the swap is out-of-the-money to the investor. The expected loss to the bank should the investor default on a standard swap after one year is 0.7% of the notional. The expected loss on the accrual range floater swap is **three times greater** at 2 % of notional. Furthermore, this more exotic swap is less amenable to credit enhancement in the form of a loss reserve fund due to the long loss tail of the swap value distribution.

Figure 2 provides the probability density functions for two bid-side swaps.

Figure 2 **Distributions of Two Year Swap Values After One Year**



The more symmetric of the two curves depicts the standard swap in which the investor pays the 6.4% market fixed rate and receives LIBOR. In the accrual range floater version, the investor pays this 6.4% rate and receives LIBOR plus 280 basis points accrued only when LIBOR is within the corridor. Many of the qualitative observations for the first example concerning the shapes of the distributions hold here as well. The downside is somewhat more limited in this case since the investor's worst scenario is that he/she ends up paying the fixed rate and receiving nothing. In the earlier swap, the investor might also have received nothing but would additionally pay (a potentially increasing) LIBOR.

The diminished downside risk is also evident in the expected losses (credit risk) of the bank transacting the swap with the investor. These values are 0.4% and 1.3% of the notional, respectively, for the standard and accrual range floater bid-side swaps. This result is consistent with the observation that a party has greater credit risk with a counterparty when it is bid-side in a swap as opposed to offer-side. The investment bank is offer-side in this example of the investor's bid-side swap.

The corridor note pdf's are similar to those of the swaps

We now consider two fixed coupon notes: a conventional note paying a 6.4% coupon and the corridor note that pays 8.5% accrued when LIBOR is within the corridor range. The probability density functions appear in Figure 3.

The distribution for the conventional bond is the more symmetric of the two curves. It is not surprising that the general shape of the corridor note distribution is similar to that of the offer-side accrual range floater swap of our first example. The corridor note is just a conventional note "swapped" with the range floater. Here we simply see the additional interest rate risk of the underlying note. The earlier observations are still valid. The investor in the corridor note will find his/her note value at less than 94% of par after one year with probability 14%.

As a final example, in Figure 4 we show the density function for the corridor-enhanced FRN in comparison with the fixed coupon note.

The distribution for this version of the corridor note is similar to that of the previous version with the exception that this distribution is somewhat more sharply peaked. This result is sensible since the coupon of this note is LIBOR plus 280 basis points accrued when LIBOR is within the corridor. Thus, it has more "character" of a conventional FRN (the coupon of which would simply be LIBOR). Floating rate notes, of course, have very little price variation. In fact, we chose not to plot the conventional FRN

Figure 3 **Distributions of Two Year Note Values After One Year**

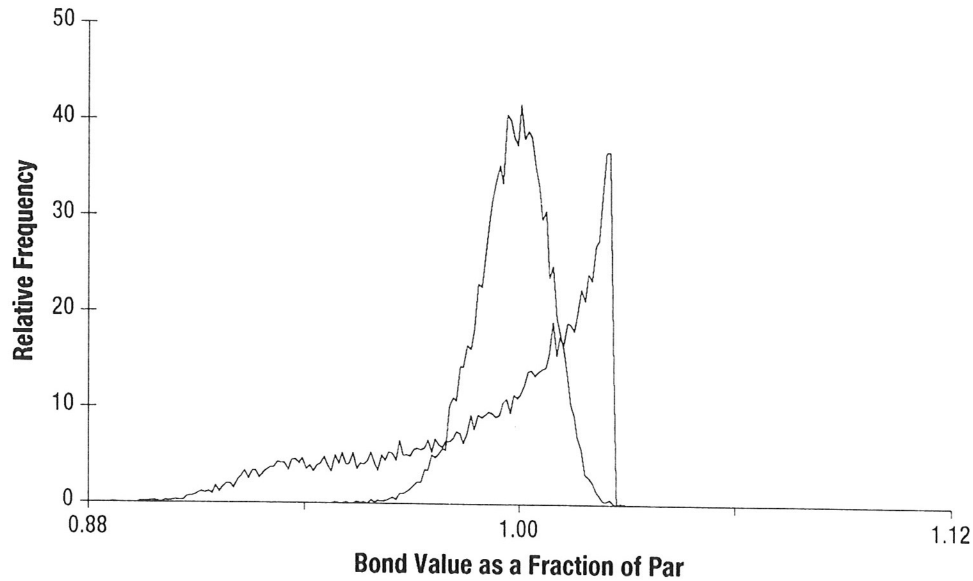
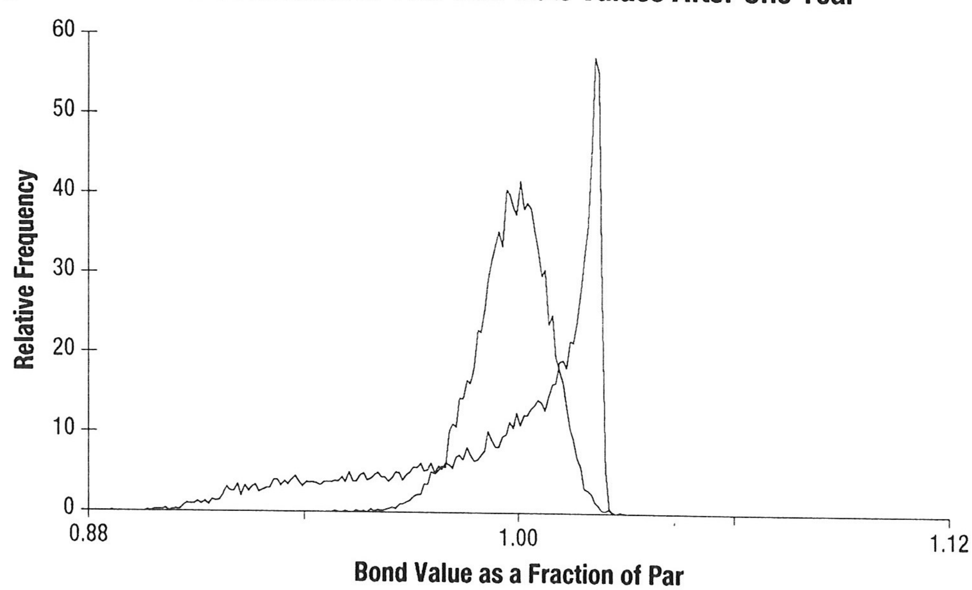


Figure 4 **Distributions of Two Year Note Values After One Year**



probability density function here since it simply shows a narrow spike at par. (For this FRN, the time at which the price variance is maximised would be just prior to a coupon payment/reset.)

As a final remark, we chose to scrutinise two-year notes and swaps, as opposed to those with longer tenor, for two reasons. We were able to leave the problem as uncluttered as possible with a single corridor specification. Also, essentially all the notes and swaps of this nature we've seen have been limited to this two year tenor/maturity. But it is certainly clear that such instruments in longer tenors would have correspondingly greater price uncertainties.

Summary

We've performed a quantitative analysis of the market and credit risks of several types of "corridor notes" and "accrual range floater swaps". We find that it is indeed the case that one may possibly enjoy higher returns, but a clear downside exists in the form of an extended tail in the oddly skewed, non-intuitive price distribution. This downside is relatively impervious to (reserve fund) credit enhancement and implies significant probability of severe market losses.

APPENDIX: Cap and Floor Valuations

Mathematical expressions for valuing conventional and digital caps and floors are widely understood and accepted in the finance community (although there do exist unresolved theoretical issues concerning interest rate options). The relevant formulae are:

$$\text{Conventional Cap}: \; zcdf * \; period * \left[F\Phi(d) - X\Phi\left(d - \sigma\sqrt{t}\right) \right]$$

$$\text{Conventional Floor}: \; zcdf * \; period * \left[-F\Phi(-d) + X\Phi\left(-d + \sigma\sqrt{t}\right) \right]$$

$$\text{Digital Cap}: \; zcdf * \; period * \; payout * \; \Phi\left(d - \sigma\sqrt{t}\right)$$

$$\text{Digital Floor}: \; zcdf * \; period * \; payout * \; \Phi\left(-d + \sigma\sqrt{t}\right)$$

There are several terms in these equations that we must define. The time until option expiration (which is generally less than the time until option payment) is t and *zcdf* is the zero coupon discount factor for the **payment** time. The period measured in years "controlled" by the cap or floor is *period.* The digital payout (*e.g.,* 8.5 %) is *payout.* F and X stand for the forward LIBOR setting at option expiration and strike rate, respectively. The character σ is the LIBOR volatility. $\Phi(x)$ is the standard normal cumulative density function

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} du \, \exp\left(-u^2/2\right)$$

and the parameter d is defined to be

$$d = \frac{\log(F/X) + \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$

where "log" denotes the natural logarithm. Further, one should multiply all option values above by the swap notional.