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Test Strategy to Minimize Semiconductor Manufacturing Expense

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Introduction

Semiconductor manufacturing's history is a story of encountering and generally overcoming engineering, manufacturing, and technological barriers. The story continues unabated today and there's little reason to doubt continued improvements in performance, density, and cost.

Many narratives omit the manufacturers' success in vaulting the "financial" barrier. To put it plainly, semiconductor manufacturing is daunting as a business challenge. Investments in plant and capital equipment are huge. Technology evolution is unrelenting. Profit margins are always pressured. Similar to the science and engineering components, effectively running this business demands genius-level discoveries and methods.

We present in this article an algorithm and discussion for minimizing processing and testing costs of a semiconductor manufacturing line. The formulation and implementation of the algorithm is necessarily an "engineering-side" activity. Yet the goal is optimization of the business. Coupling this observation with the critical role of the Chief Financial Officer as the "owner" of financial statement information, we realize that Finance and Engineering must collaborate in the optimal testing platform we propose. Neither Engineering nor Finance alone have the practical ability to optimize testing costs in this manner.

In the early sections of this article we take the Engineering side. That is, we describe the costs of the manufacturing line as engineers understand them in daily operation. We formulate the optimal strategy problem. With some preceding discussion of special cases and approximations, we then find an exact solution for the optimal strategy and provide several simple examples. As we will reiterate several times, our examples do not employ cost data or any other information of a specific manufacturer.

Late in the article, though, we note that the most challenging part of this exercise resides in the domain of Finance. As we explain, key components of the firm's financial statements are the inputs to the optimization algorithm. Estimating these inputs is "the hard part" of this project. It would be ineffective, and likely even counterproductive, for Engineering to estimate the critical inputs without the knowledge and participation of Finance. Specifically, if Engineering employs inputs that are inconsistent with the manufacturer's financial statements, then any "optimization" that Engineering achieves will not optimize the firm's performance as measured in the financial statements.

Not surprisingly, our view is that Engineering and Finance *should* collaborate to the benefit of all. This study contributes both a proposal to optimize engineer testing decisions on the production line and a management directive to create a stronger business.

Mapping the Costs of the Manufacturing Line

We decompose the manufacturing process from initial polished Silicon substrate to completion of the "lot" of finished wafers into N critical processing steps. Typical steps are oxidation, lithography, depositions of insulators and conductors, etching, ion implantation, impurity diffusion and annealing.[1] The tests and inspections that technicians and engineers may perform following each of these processing steps determine the success and quality of each operation. Such reviews include measurements of layer thicknesses and resistivities as well as visual and scanning electron microscope observations of line widths, edges and particulates.

We stipulate that the result of performing the test after step j is "pass-fail." When the wafer lot passes the test, it proceeds to the next processing step. If the lot fails the test, the engineer terminates all further processing and discards the entire lot. This action is sensible in that it treats test failure at step j as indication that ultimate lot yield – after all processing – would be

[1] An abundance of references are available for each of these processing steps. See, for example, A.S.Grove, *Physics and Technology of Semiconductor Devices*, Wiley, 1967; and J.M.Pimbley, M.Ghezzo, H.G.Parks, and D.M.Brown, *Advanced CMOS Process Technology*, Academic Press, 1989.

zero or near-zero if the lot continued through all further steps. Termination of the lot at step j saves the expense of all subsequent processing.

We further assume that the test procedure of each step is accurate. That is, failure of the test implies ultimate zero or near-zero die yield as we state above *and* lots with positive test outcome have statistically significant higher yields with respect to potential defects measurable at the specific step. From the aggregated data of the manufacturing line, we infer the test failure probability p_j for the processing step j.

If the cost of performing each test were zero, then the optimal test strategy would be trivial: simply perform all the tests. As long as the tests are accurate, it is better to know immediately after step j if a defect of any kind occurred at this step so that the line will waste no further resources on the damaged lot. But the challenge is that each test operation does have a cost which we designate as ζ_j (cost of test performed after processing step j). This cost ζ_j consists of both direct costs for testing equipment and personnel as well as indirect costs such as idle time for the next step's processing equipment and personnel.

Finally, let us define the *cost of each lot* after processing step j, excluding test costs, as V_j. We are tempted to call this a "value" rather than a "cost" of the lot since the former term has a more positive connotation. But our purpose is to track the cost of each lot as it proceeds through manufacturing and to save cost with the optimal trade-off between testing, which adds cost, and scrapping a defective lot prior to completion, which reduces cost. In addition, our values V_j, augmented with test costs, should coincide with the financial reporting of the business (*e.g.*, the value of inventory as an asset on the balance sheet and the cost-of-goods-sold – "COGS" – as a line item on the income statement).[2]

[2] We are aware of the earlier work of N.Li, L.Zhang, M.Zhang, and L.Zheng, "Applied Factory Physics Study on Semiconductor Assembly and Test Manufacturing," *IEEE Xplore Conference for Semiconductor Manufacturing*, 2005. The goals of this study differed from ours.

Formulation of the Optimal Test Strategy

To add clarity to our definitions of the prior section, V_0 is the procurement cost of the lot of polished Silicon substrates and all internal "handling" costs to prepare the lot to begin the long list of defined processing steps. If a fiscal quarter ends on the day that this lot begins processing, then the balance sheet inventory value of the lot for this quarter should be V_0. Upon completion of the first critical processing step, the lot value will be V_1 – which is equivalent to the earlier value V_0 plus the total cost (materials, equipment, labor, *et cetera*) of this first processing step. If the engineer chooses to test the lot following the first processing step, there is probability p_1 of getting a failing test result which then scraps the lot. With probability $1 - p_1$, the lot will pass the test and proceed to processing step two with new total lot cost of $V_1 + \zeta_1$. Conversely, if the engineer chooses *not* to test the lot, it will simply proceed to step two and maintain the lot cost of V_1.

The total production cost depends on the parameters we've defined as well as the engineer's choices for testing at each of the *N* critical processing steps. Let's write w_j for the testing choice after step *j*. This w_j is either zero ("do not test") or one ("perform the test"). The total cost is:

$$Total\ Cost\ =\ V_1 + \sum_{j=1}^{N-1} S_{j-1}\left[\left(1-p_j\right)^{w_j}\left(V_{j+1}-V_j\right) +\ \zeta_j w_j\right]\quad (1a)$$

$$S_j\ =\ \prod_{k=1}^{j}(1-p_k)^{w_k}\quad (1b)$$

In equation (1b), S_j is the "survival probability" of the lot after testing, or after the decision not to test at this point, following step *j*. The first survival probability, S_0, is equal to one.

As calibration, let's note the special case in equations (1a-b) if the engineer chooses *no testing* at each step. That means all the $w_j = 0$. Plugging in these zero values, all the $S_j = 1$ and the total cost of the lot is V_N. We expect this result since our definition of V_j is the cost of the lot after processing step *j* in the absence of all test costs.

As a second special case for (1a-b), imagine the engineer performs *every* possible test. That makes all the $w_j = 1$. Adding the further gross simplification that all test failure probabilities p_j are zero, we find that the (1b) survival probability $S_j = 1$ again (*i.e.*, the engineer scraps no lots since there are no test failures in this special case). Now the total lot cost in (1a) is V_N plus the sum of all the test costs ζ_j. This sensible result merely states that the lot incurs the costs of each processing step, aggregating to V_N, since there are no test failures (due to the $p_j = 0$ assumption). To this total processing cost V_N we must then add the costs ζ_j of all tests to tally the total lot cost.

If in this last special case we remove the assumption that $p_j = 0$, then we cannot deduce by inspection the survival probability S_j and total lot cost. We do know that $S_j < 1$, which means that the total lot cost will not include the full processing cost due to non-zero probability of scrapping the lot prior to completion of all *N* processing steps.

Solution for the Optimal Test Strategy

Our challenge is to find the values of the $N - 1$ individual w_j that minimize the total production cost of equation (1a). Upon inspection of (1a), one sees an immediate "possible answer" employing the property that the survival probabilities and bracketed terms are all greater than zero. This "possible answer" emerges by choosing the w_j so that the bracketed term in (1a) is always the smaller value. That is, we perform the test (*i.e.*, set $w_j = 1$) at processing step *j* if

$$\zeta_j < p_j\left(V_{j+1} - V_j\right) \quad (2)$$

If the inequality in (2) is true, then we set $w_j = 1$ and perform the test at this step. But if the inequality of (2) is false, then we put $w_j = 0$ and *do not* perform the test. This "possible answer," as we call it, is simple and intuitive. Simple, intuitive decision logic is highly desirable in a production line environment.

Unfortunately, this potential solution (2) for (1a-b) is not accurate as we show in subsequent examples. Simply minimizing the bracketed terms in the summation of (1a) ignores the role of the S_{j-1} multiplier. Equation (2) is helpful in that it shows the common sense outcome that the engineer should always perform a test with zero cost since, with $\zeta_j = 0$, the inequality (2) will always be satisfied as long as $p_j > 0$. Yet the overall inadequacy of this approximation quickly becomes apparent in example calculations.

As it happens, we *are* able to solve exactly for the set of w_j that minimizes the cost equations (1a-b). The key insight is to define the indexed parameter T_j as "the cost of remaining processing after step *j plus* the cost of optimal testing at all steps at and after *j*." By inspection of (1a-b), we find that the backward recursion sequence of the T_j creates an identical representation of the "total cost," which will be $V_1 + T_1$, with these rules:

$$T_N = 0\,;$$

$$T_j = \left(1 - p_j\right)^{w_j}\left(V_{j+1} - V_j + T_{j+1}\right) + \zeta_j w_j\,, \qquad j = N-1, \ldots, 2, 1 \;(3)$$

The solution procedure is to begin with $j = N - 1$:

$$T_{N-1} = (1 - p_{N-1})^{w_{N-1}}\,(V_N - V_{N-1}) + \zeta_{N-1} w_{N-1}$$

We simply determine whether setting w_{N-1} to 0 or to 1 provides the minimum T_{N-1}. This procedure then gives definite values for w_{N-1} and T_{N-1}. Next we march (backward) to $j = N - 2$ and determine w_{N-2} and T_{N-2} from the similar relation

$$T_{N-2} = (1 - p_{N-2})^{w_{N-2}}\,(V_{N-1} - V_{N-2} + T_{N-1}) + \zeta_{N-2} w_{N-2}$$

We continue the backward recursion until we reach $j = 1$ and thus determine w_1 and T_1 in this last step. This complete set of w_j is the optimal test strategy. The quantity $V_1 + T_1$ is the total cost including optimal testing.

To recap, we have replaced equations (1a-b), a series summation and a series product, representing the total cost of processing and testing with equation (3), a backward recursion. The two forms are mathematically equivalent. The backward recursion form, however, permits a simple step-by-step optimization of the w_j.

This backward recursion optimization is unavoidably a numerical calculation. But there still exists an intuitive meaning. Instead of equation (2) for the "possible answer" approximation we discussed earlier, the exact solution takes the form of the rule to set $w_j = 1$ and perform the test when

$$\zeta_j < p_j\left(V_{j+1} - V_j + T_{j+1}\right) \quad (4)$$

We have the difficulty of requiring the numerical calculation of equation (3) to learn the value of T_{j+1}. But this form (4) shows the understandable behaviour that performing the test becomes more likely as either the cost ζ_j decreases or the test failure probability p_j increases.

Simple Examples of the Optimal Test Strategy

Let's consider a few simple examples to demonstrate both the approximation of equation (2) and the exact result of equations (3) and (4). We specify $N = 20$ critical processing steps with equal step costs, test costs, and test failure probabilities at each step. Clearly this is unrealistic since true semiconductor manufacturing steps vary widely in complexity and cost. See Figure 1.

Critical Process Step	Cost of Process Step ($ thousand)	Cost of Test ($ thousand)	Probability of Test Failure	Test? (Simple)	Test? (Exact)	Remaining Cost ($ thousand)
0	50.0					
1	20.0	5.0	4.0%	0	1	319.7
2	20.0	5.0	4.0%	0	1	307.8
3	20.0	5.0	4.0%	0	1	295.5
4	20.0	5.0	4.0%	0	1	282.6
5	20.0	5.0	4.0%	0	1	269.1
6	20.0	5.0	4.0%	0	1	255.1
7	20.0	5.0	4.0%	0	1	240.5
8	20.0	5.0	4.0%	0	1	225.4
9	20.0	5.0	4.0%	0	1	209.5
10	20.0	5.0	4.0%	0	1	193.1
11	20.0	5.0	4.0%	0	1	175.9
12	20.0	5.0	4.0%	0	1	158.0
13	20.0	5.0	4.0%	0	1	139.4
14	20.0	5.0	4.0%	0	0	120.0
15	20.0	5.0	4.0%	0	0	100.0
16	20.0	5.0	4.0%	0	0	80.0
17	20.0	5.0	4.0%	0	0	60.0
18	20.0	5.0	4.0%	0	0	40.0
19	20.0	5.0	4.0%	0	0	20.0
20	20.0	5.0	4.0%			0.0

Figure 1

Just to emphasize, the Figure 1 example is highly simplified and the specific values are neither realistic nor sourced from a real entity. Of the seven columns, the fifth shows the result of the "possible answer" of equation (2) for testing the lot after each processing step. The zero values of that column depict "no testing" at any step. Such values would change depending on changes to the input information and data of the four columns to the left.

In the two columns to the far right of Figure 1 we show calculation results for and w_j and T_j, respectively. These are the exact solutions of equations (3) and (4) which we are able to impose with simple spreadsheet formulas for the backward recursion. Note the interesting result that testing after each step is optimal until the thirteenth critical processing step. Thereafter, the algorithm recommends no further testing. It is the reduced remaining cost of lot processing that makes the later stage testing sub-optimal. Stated differently, there is less savings due to scrapping a lot as one progresses further through the fabrication sequence. Hence, the manufacturer is able to reduce costs by deleting tests near the end of the processing sequence.

We don't expect all examples to have the simple structure of "test at all early processing steps and then cease testing at some point" since it is highly likely that steps that are especially costly or that have combinations of inexpensive test procedures and high-probability test failures will distort the simple pattern. But we do expect all examples to show the *tendency* that testing is more sensible early in the manufacturing line rather than late in the line.

In Figure 2 we make modest, arbitrary adjustments to the test cost and test failure probability. Results are similar but not identical to those of Figure 1. In particular, the exact solution shows skipping the single test after the tenth processing step is optimal due, apparently, to somewhat higher test cost and lower failure probability.

Critical Process Step	Cost of Process Step ($ thousand)	Cost of Test ($ thousand)	Probability of Test Failure	Test? (Simple)	Test? (Exact)	Remaining Cost ($ thousand)
0	50.0					
1	20.0	3.0	3.0%	0	1	320.0
2	20.0	5.0	4.0%	0	1	306.8
3	20.0	7.0	5.0%	0	1	294.4
4	20.0	7.0	3.0%	0	1	282.5
5	20.0	5.0	4.0%	0	1	264.1
6	20.0	3.0	5.0%	0	1	249.9
7	20.0	3.0	3.0%	0	1	239.9
8	20.0	5.0	4.0%	0	1	224.2
9	20.0	7.0	5.0%	0	1	208.3
10	20.0	7.0	3.0%	0	0	191.9
11	20.0	5.0	4.0%	0	1	171.9
12	20.0	3.0	5.0%	0	1	153.9
13	20.0	3.0	3.0%	0	1	138.8
14	20.0	5.0	4.0%	0	0	120.0
15	20.0	7.0	5.0%	0	0	100.0
16	20.0	7.0	3.0%	0	0	80.0
17	20.0	5.0	4.0%	0	0	60.0
18	20.0	3.0	5.0%	0	0	40.0
19	20.0	3.0	3.0%	0	0	20.0
20	20.0	5.0	5.0%			0.0

Figure 2

As just one further example, we maintain the previous input information with the exception of reducing all test costs by 80%. Figure 3 shows, as one might expect, that the test cost reduction encourages more testing.

Critical Process Step	Cost of Process Step ($ thousand)	Cost of Test ($ thousand)	Probability of Test Failure	Test? (Simple)	Test? (Exact)	Remaining Cost ($ thousand)
0	50.0					
1	20.0	0.6	3.0%	0	1	274.2
2	20.0	1.0	4.0%	0	1	262.0
3	20.0	1.4	5.0%	0	1	251.9
4	20.0	1.4	3.0%	0	1	243.7
5	20.0	1.0	4.0%	0	1	229.8
6	20.0	0.6	5.0%	1	1	218.3
7	20.0	0.6	3.0%	0	1	209.2
8	20.0	1.0	4.0%	0	1	195.0
9	20.0	1.4	5.0%	0	1	182.1
10	20.0	1.4	3.0%	0	1	170.2
11	20.0	1.0	4.0%	0	1	154.0
12	20.0	0.6	5.0%	1	1	139.4
13	20.0	0.6	3.0%	0	1	126.1
14	20.0	1.0	4.0%	0	1	109.4
15	20.0	1.4	5.0%	0	1	92.9
16	20.0	1.4	3.0%	0	1	76.3
17	20.0	1.0	4.0%	0	1	57.3
18	20.0	0.6	5.0%	1	1	38.6
19	20.0	0.6	3.0%	0	0	20.0
20	20.0	1.0	5.0%			0.0

Figure 3

As a comment that is valid for all three preceding figures, we note that the most important aspect of the "Remaining Cost" column to the far right is the first value T_1. In Figure 3, for example, this value is $274,200. Given the (unrealistic) values of input information – in particular the $20,000 cost for each of 20 critical processing steps – the total cost of production *in the absence of testing* is $400,000. That is, with no testing, the engineer would push all lots through the entire manufacturing sequence and this cost of $20,000 multiplied by 20 steps gives the $400,000. But with optimal testing as per this algorithm, the production cost, $V_1 + T_1$, is the much lower $294,200.

Importance of Adaptive Parameters

While our solution to the "optimal test strategy" is exact for the manner in which we posed this problem in equations (1a-b), the formulation itself has numerous idealizations and simplifications. We assume, for example, the ideal case in which a test result, whether "pass" or "fail," is always accurate. We assume the engineer knows all processing step costs, test costs, and test failure probabilities and further that such costs and probabilities remain fixed. Individual firms may find it beneficial to relax one or several of these assumptions.

To take just one possibility, imagine that a lot has just finished a critical processing step and the engineer must choose whether or not to perform the available test at this point in the line. Given the fixed cost of the step and other parameters, let's say the "optimal test strategy" algorithm recommends skipping the test. Imagine, though, that a component of this test's cost is the idle time it would enforce on the equipment (lithography tool, ion implanter, *et cetera*) of the next critical processing step. But this idle time cost element depends on the status of the equipment. That is, if the current lot must wait in queue for availability of the equipment of the next step, then there may be no "cost" of the testing associated with equipment idle time since the equipment is not idle and the lot processing is delayed whether or not it undergoes testing.

Stated differently, one reasonably expects that the costs of both processing and testing at various steps may depend on the "states" of equipment, testing stations, and other lots that compete for resources. Hence, rational, real-time adjustments to estimated costs would create an improved, dynamic formulation for "optimal test strategy."

True Value of this Optimization Model is the Institutional Learning and Collaboration

The truly challenging aspect of this optimization exercise is not the evaluation of the algorithm of equation (3). What is most challenging is learning, recording, and *agreeing* within the institution the values of processing costs V_j, testing costs ζ_j, and failure probabilities p_j. Let's remove the last of these, p_j, from consideration since it is far easier to retain and analyse manufacturing line data relevant to these probabilities than it is to evaluate processing and testing costs.

Our thesis in this research is that it is well within the purview of the Chief Financial Officer (CFO) and his/her Finance team to estimate these costs. The CFO and team prepare the firm's financial statements. It is impossible to overstate the importance of accuracy and completeness of the financial statements to investors (both shareholders and lenders), customers, vendors, and other stakeholders including employees, taxing and regulating authorities, auditors, and even competitors. Beyond the role of financial statements in reporting to these constituencies, the statements are critical for management's own decision-making and stewardship of the business.

The value of "Inventory" on a semiconductor manufacturer's balance sheet, one of the financial statements, is generally the cost of both fully processed, but unsold, lots and current cost of unfinished lots.[3] As an asset, inventory is a direct positive contributor to the "net worth" of the business. In this sense, a high level of inventory is good. But to the CEO and others, high inventory *may* also signal risk to the business, inefficiency in managing the business, or failing sales. Similarly, low inventory can have both good and bad connotations.

Processing and testing costs also feed "cost of goods sold" (COGS), another accounting element.[4] COGS appears as an expense on the income statement (another of the collection of reports that constitute "the financial statements"). COGS and inventory bear a direct relationship because it is the

[3] See explanation of Inventory, *Accounting-Simplified.com*, at http://www.accounting-simplified.com/financial-accounting/accounting-for-inventory/ .

[4] See explanation of COGS, *Accounting Coach*, at https://www.accountingcoach.com/blog/cost-of-goods-sold-2 .

product in inventory that becomes the "goods sold." Hence, when a firm sells some inventory, the inventory line on the balance sheet decreases and the same amount appears as COGS on the income statement. The price the firm receives for selling this inventory is "sales revenue." Of course, the difference between sales revenue and COGS, the "gross profit," is fundamental to business performance.

We have multiple objectives in providing these brief explanations of the accounting concepts of inventory and COGS. First, as these are financial statement entries, it is the CFO who bears primary responsibility and incentive to estimate the two elements accurately. Second, the goal of the CFO and Chief Executive Officer (CEO) is not merely "good reporting" but also achieving excellence in managing the business. As we noted, inventory levels say much about the quality, efficiency, and risk of the firm. COGS directly impacts profitability. Excellence in management requires deep expertise in all critical components of the financial statements including inventory and COGS.

Third, the impetus of this research is the identification of an algorithm for optimal testing in a semiconductor manufacturing line. Clearly, the owner and beneficiary of this worthy goal is not simply the engineering staff. While it is the engineers who make optimal testing decisions that effectively manage the inventory and minimize COGS, the true "owner" of this activity is the CFO and, by extension, the CEO.

Fourth and finally, we reach the summit of this communication. We began this section by writing that the challenging aspect of optimizing testing on the manufacturing line is learning, recording, and *agreeing* within the institution the values of processing costs V_j and testing costs ζ_j. What the firm needs is a strong collaboration between Finance and Engineering to make this happen. Finance already bears the fiduciary and management responsibility to estimate and report processing and testing costs in the financial statements. Yet it's a daunting task to make *truly* good estimates. Mediocre estimates of costs will precipitate mediocre, essentially worthless, optimization attempts. It is through the strength of Finance (attention to detail, discipline inherent in its obligations to auditors and the Board of

Directors, unquestioned support of management given the CFO's ownership) and the strength of Engineering (technical expertise, presence on the line, daily accountability for die yield and equipment up-time, and analytical intuition to identify poor cost estimates) in collaboration that the firm will achieve best processing and testing cost estimates.